Elektromagnetism, TFYA13, 2013-08-21

1) a)
$$\underline{\underline{\bar{A}(\bar{r})}} = \int_{\circledast} \frac{\mu_0 \bar{J}(\bar{r}')\, dQ'}{4\pi|\bar{r}-\bar{r}'|} = \int_a^b\int_0^\pi \frac{\mu_0 J_0 \hat{\phi}}{4\pi (R')^2} R' d\phi' dR' =$$

$$= \frac{\mu_0 J_0}{4\pi}\int_a^b\int_0^\pi \frac{\cos\phi'\hat{y} - \sin\phi'\hat{x}}{R'}\, d\phi' dR' =$$

$$= \frac{\mu_0 J_0}{4\pi}\int_a^b \Big[\underset{\substack{\uparrow\\ 0-0}}{\sin\phi'\hat{y}} + \underset{\substack{\uparrow\\ -1-1}}{\cos\phi'\hat{x}}\Big]_0^\pi \frac{dR'}{R'} = \frac{\mu_0 J_0}{2\pi}\Big[-\ln R'\Big]_a^b \hat{x}$$

$$\underline{\underline{= \frac{-\mu_0 J_0}{2\pi}\ln b/a\, \hat{x}}}$$

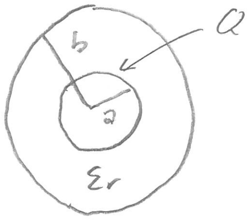
b)
$$\underline{\underline{\bar{B}(\bar{r})}} = \int_{*} \frac{\mu_0 \bar{J}(\bar{r}')\times(\bar{r}-\bar{r}')}{4\pi|\bar{r}-\bar{r}'|^3}\, dQ' = \int_a^b\int_0^\pi \frac{\mu_0 J_0 \overbrace{\hat{\phi}'\times(\bar{0}-R'\hat{R}')}^{\hat{z}}}{4\pi (R')^4} R' d\phi' dR' =$$

$$= \frac{\mu_0 J_0}{4}\int_a^b \frac{dR'}{(R')^2}\hat{z} = \frac{\mu_0 J_0}{4}\left(\frac{-1}{b}+\frac{1}{a}\right)\hat{z} = \underline{\underline{\frac{\mu_0 J_0}{4}\cdot\frac{b-a}{ab}\hat{z}}}$$

Svar: a) Magnetiska vektorpotentialen i origo är $\underline{\underline{\bar{A} = \frac{-\mu_0 J_0}{2\pi}\ln b/a\, \hat{x}}}$

b) Magnetiska flödestätheten i origo är $\underline{\underline{\bar{B} = \frac{\mu_0 J_0}{4}\cdot\frac{b-a}{ab}\hat{z}}}$

2)



Ansätt laddn. Q på inre sfären.

Sym. $\Rightarrow \bar{D} = D(r)\hat{r}$

Gauss sats $\oint \bar{D}\cdot d\bar{S} = Q_{fri,innes}$ $\Big\} \Rightarrow 4\pi r^2 D(r) = Q$

$$\Rightarrow \bar{D} = \frac{Q}{4\pi r^2}\hat{r} = \varepsilon_0 \varepsilon_r \bar{E} \Rightarrow \bar{E} = \frac{Q}{4\pi \varepsilon_0 \alpha r^3}\hat{r} \; ; a<r<b$$

$$W_e = \underset{\text{"Hela } R^3\text{"}}{\int} \frac{1}{2}\bar{D}\cdot\bar{E}\, d\tau = \int_a^b\int_0^{2\pi}\int_0^{\pi} \frac{1}{2}\frac{Q^2}{(4\pi)^2 \varepsilon_0 \alpha r^5} r^2 \sin\theta\, d\theta\, d\phi\, dr +$$

$$+ \underset{\substack{\uparrow \\ \text{Inga fält} \\ \text{utanför}}}{0} = \frac{Q^2}{4\cdot 4\pi\cdot\varepsilon_0\cdot\alpha}\left[\frac{-1}{r^2}\right]_a^b = \frac{Q^2}{4\cdot 4\pi\cdot\varepsilon_0\alpha}\left(\frac{1}{a^2} - \frac{1}{b^2}\right) \quad ①$$

Hitta Q uttryckt i U:

$$U = \int_{Ref}^{Alt} -\bar{E}\cdot d\bar{l} = \int_b^a \frac{-Q}{4\pi\varepsilon_0 \alpha r^3}\hat{r}\cdot\hat{r}\, dr = \frac{Q}{2\cdot 4\pi\varepsilon_0\alpha}\left[\frac{1}{r^2}\right]_b^a =$$

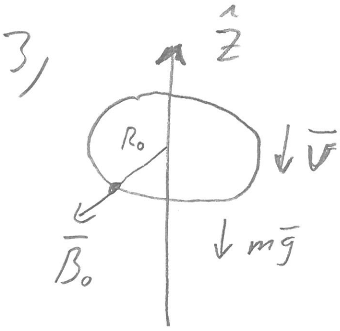
$$= \frac{Q}{2\cdot 4\pi\varepsilon_0\alpha}\left(\frac{1}{a^2} - \frac{1}{b^2}\right) \Rightarrow Q = 2\cdot 4\pi\varepsilon_0\alpha\frac{a^2b^2}{b^2-a^2}U \quad ②$$

$$② \text{ i } ① \Rightarrow \underline{\underline{W_e = \frac{UQ}{2} = 4\pi\varepsilon_0\alpha\frac{a^2b^2}{b^2-a^2}U^2}}$$

<u>Svar: Kondensatorns energi är:</u>

$$\underline{\underline{W_e = 4\pi\varepsilon_0\alpha\frac{a^2b^2}{b^2-a^2}U^2}}$$

3)



$\downarrow \bar{v} = -v\hat{z}$

Stoppar in ett explicit "−" så att $v > 0$.

$$\mathcal{E}_{ems} = \int (\bar{v} \times \bar{B}) \cdot d\bar{l} =$$

$$= \int_0^{2\pi} (-v\hat{z} \times B_0\hat{R}) \cdot R_0\hat{\phi}\, d\phi = -2\pi v B_0 R_0$$

Den inducerade spänningen ger upphov till en ström i ringen, $i = \mathcal{E}_{ems}/R_\Omega$.

Magnetisk kraft på ringen:

$$\bar{F}_m = \int I\, d\bar{l} \times \bar{B} = \int_0^{2\pi} \frac{\mathcal{E}_{ems}}{R_\Omega} \overbrace{\hat{\phi} R_0\, d\phi \times B_0 \hat{R}}^{\hat{z}} =$$

$$= \frac{\mathcal{E}_{ems}}{R_\Omega} R_0 B_0 \cdot 2\pi (-\hat{z}) = \frac{(2\pi R_0 B_0)^2}{R_\Omega} v\, \hat{z}$$

Tyngdkraft på ringen: $\bar{F}_g = mg(-\hat{z})$

Kraftjämvikt: $\bar{F}_m + \bar{F}_g = \bar{0} \Rightarrow \frac{(2\pi R_0 B_0)^2}{R_\Omega} v = mg$

$$\Rightarrow \underline{\underline{v = \frac{mg R_\Omega}{(2\pi R_0 B_0)^2}}}$$

Svar: Ringens jämviktsfart blir

$$\underline{\underline{v = \frac{mg R_\Omega}{(2\pi R_0 B_0)^2}}}$$

4) Betrakta geometrin som en superposition av två cylinderladdningstätheter. En med laddn. täthet ρ, radie $3a$, och symmetriaxel genom origo. Den andra med laddn. täth. $-\rho$, radie a, och sym. axel genom $x=a$.

För var och en av cylinderladdn. tätheterna gäller cyl. sym. $\Rightarrow \bar{D} = D(R)\hat{R}$. Gauss sats $\oint_S \bar{D}\cdot d\bar{S} = Q_{\text{fri,innes}}$ $\Big\} \Rightarrow$

$$2\pi R L\, D(R) = \pi R^2 L \rho \Rightarrow \bar{D} = \frac{\rho R}{2}\hat{R} = \frac{\rho R}{2}\cdot\frac{x\hat{x}+y\hat{y}}{R} \Rightarrow$$

$$\bar{D} = \frac{\rho}{2}(x\hat{x}+y\hat{y}) \text{ inuti resp. cyl. laddn.}$$

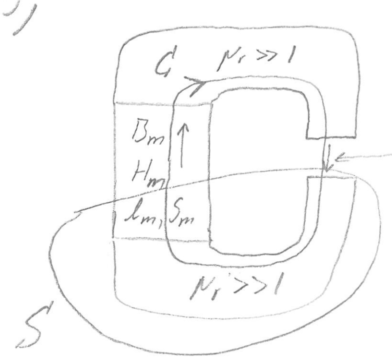
$$\bar{D}_{tot} = \underset{\substack{\uparrow\\ \text{Från}\\ \text{stora}}}{\bar{D}_{3a}} + \underset{\substack{\uparrow\\ \text{Från}\\ \text{lilla}}}{\bar{D}_{a}} = \frac{\rho}{2}(x\hat{x}+y\hat{y}) - \frac{\rho}{2}\Big[\underset{\substack{\uparrow\\ \text{Symmetriaxeln}\\ \text{genom } x=a \text{ i.s.f.}\\ \text{genom origo, } x=0.}}{(x-a)}\hat{x} + y\hat{y}\Big] =$$

$$= \frac{\rho a}{2}\hat{x} = \varepsilon_0 \bar{E} \Rightarrow \underline{\underline{\bar{E} = \frac{\rho a}{2\varepsilon_0}\hat{x}}}$$

Svar: Elektriska fältstyrkan i hålet är

$$\underline{\underline{\bar{E} = \frac{\rho a}{2\varepsilon_0}\hat{x}}}$$

5)



$$\oint_C \bar{H}\cdot d\bar{l} = I_{omsl.fri} \Rightarrow$$

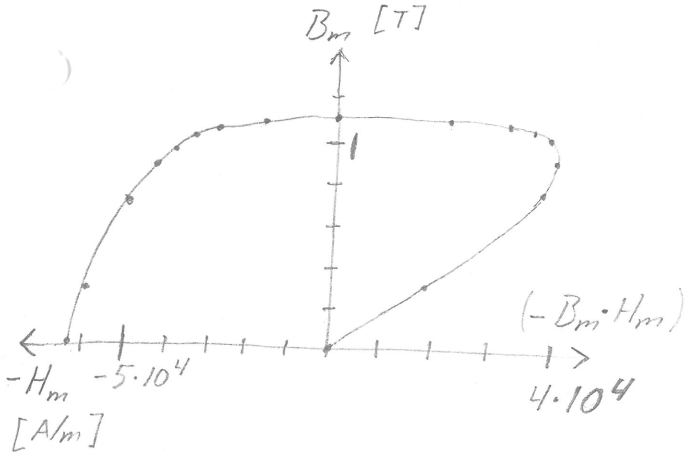
$$\underbrace{\int \bar{H}_m\cdot d\bar{l}}_{H_m l_m} + \underbrace{\int \bar{H}_p\cdot d\bar{l}}_{\approx 0} + \underbrace{\int \bar{H}_l\cdot d\bar{l}}_{H_l l_l} + \underbrace{\int \bar{H}_p\cdot d\bar{l}}_{\approx 0} = 0 \Rightarrow$$

$$H_m l_m + H_l l_l = 0 \quad ①$$

$$\oint_S \bar{B}\cdot d\bar{S} = 0, \quad \Phi = \int \bar{B}\cdot d\bar{S} \Rightarrow \Phi_m = \Phi_l \Rightarrow B_m S_m = B_l S_l \quad ②$$

Det exklusiva magnetiska materialets volym blir:

$$S_m l_m \overset{①,②}{=} \frac{B_l}{B_m} S_l \cdot \frac{H_l}{-H_m} l_l = \frac{B_l H_l S_l l_l}{-B_m H_m}$$ dvs min då $-B_m H_m$ max.



Ur graf $-B_m H_m$

max då $\begin{cases} B_m = 0,88 \\ H_m = -4,5\cdot 10^4 \end{cases}$

OBS: { Olika saker på pos. o neg. x-axlarna.

Från ②: $S_m = \frac{B_l}{B_m} S_l = \frac{1,2}{0,88}\cdot 4,0 \approx 5,5\,cm^2$

Från ①: $l_m = -\frac{H_l}{H_m} l_l = -\frac{B_l}{\mu_0 H_m} l_l = +\frac{1,2}{4\pi\cdot 10^{-7}\cdot 4,5\cdot 10^4}\cdot 0,5 \approx 11\,cm$

Svar: $S_m = 5,5\,cm^2$, $l_m = 11\,cm$