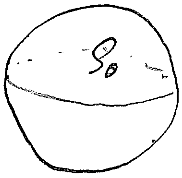
Tentamen i Elektromagnetism, TFYA13, 2012-01-14

1)



Sfärisk symmetri $\Rightarrow$ $\bar{D} = D(r)\hat{r}$

Gauss sats $\oint_S \bar{D}\cdot d\bar{S} = Q_{fri,\,inneslutet}$ $\Rightarrow$

Välj ytan S som en sfär, radie r

$$\oint_S \bar{D}\cdot d\bar{S} = \oint_S D(r)\hat{r}\cdot\hat{r}\,dS = D(r)\oint_S dS = 4\pi r^2 D(r) = Q_{fri,\,inneslutet}$$

$$Q_{fri,\,inneslutet} = \int_\tau \rho\, d\tau = \begin{cases} \rho_0 \dfrac{4\pi r^3}{3} & 0 \le r \le a \\ \rho_0 \dfrac{4\pi a^3}{3} & a \le r \end{cases} \Rightarrow$$

$$\bar{D} = \begin{cases} \dfrac{\rho_0 r}{3}\hat{r} & 0 \le r \le a \\ \dfrac{\rho_0 a^3}{3r^2}\hat{r} & a \le r \end{cases} \Rightarrow \bar{E} = \frac{\bar{D}}{\varepsilon_0\varepsilon_r} = \begin{cases} \dfrac{\rho_0 r}{3\varepsilon_0[1+\alpha(r/a)^2]}\hat{r} & 0 \le r \le a \\ \dfrac{\rho_0 a^3}{3\varepsilon_0 r^2}\hat{r} & a \le r \end{cases}$$

$$\underline{\underline{U}} = \int_{Ref}^{Akt} -\bar{E}\cdot d\bar{l} = \int_0^a \frac{-\rho_0 r}{3\varepsilon_0[1+\alpha(r/a)^2]}\,dr + \int_a^{10a} \frac{-\rho_0 a^3}{3\varepsilon_0 r^2}\,dr =$$

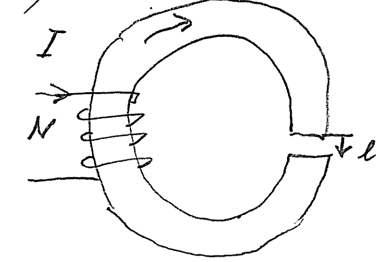
$$= \left[\frac{-\rho_0 a^2}{3\varepsilon_0 2\alpha}\ln(1+\alpha(r/a)^2)\right]_0^a + \left[\frac{\rho_0 a^3}{3\varepsilon_0 r}\right]_a^{10a} =$$

$$= \frac{-\rho_0 a^2}{6\alpha\varepsilon_0}\ln(1+\alpha) + \frac{\rho_0 a^2}{3\varepsilon_0 10} - \frac{\rho_0 a^2}{3\varepsilon_0} =$$

$$= \underline{\underline{\frac{-\rho_0 a^2}{3\varepsilon_0}\left[\frac{9}{10} + \frac{1}{2\alpha}\ln(1+\alpha)\right]}}$$

Svar: $\underline{\underline{U = -\dfrac{\rho_0 a^2}{3\varepsilon_0}\left[\dfrac{9}{10} + \dfrac{1}{2\alpha}\ln(1+\alpha)\right]}}$

2)



$$\oint_C \bar{H}\cdot d\bar{l} = I_{tri.oms.} \Rightarrow H_m L + H_l l = NI \quad ①$$

$$\bar{B} = \mu_0\mu_r \bar{H} \Rightarrow B_m = \mu_0\mu_r H_m \quad ② \qquad B_l = \mu_0 H_l \quad ③$$

$$\Phi = \int \bar{B}\cdot d\bar{S} \Rightarrow \Phi_m = B_m S_m \quad ④ \qquad \Phi_l = B_l S_l \quad ⑤$$

$$0 = \oint \bar{B}\cdot d\bar{S} \Rightarrow \Phi_m = \Phi_l \equiv \Phi \quad ⑥ \qquad S_m = S_l = A \quad ⑦$$

$$\therefore \; B_m = \Phi/A \;, \; B_l = \Phi/A \;, \; H_m = \frac{\Phi}{\mu_0\mu_r A} \; ⑧ \;, \; H_l = \frac{\Phi}{\mu_0 A} \; ⑨$$

$$W_m = \int_{\text{"Hela } R^3\text{"}} \frac{1}{2}\bar{B}\cdot\bar{H}\, d\tau = \underbrace{\frac{1}{2}\frac{\Phi}{A}\cdot\frac{\Phi}{\mu_0\mu_r A} A\cdot L}_{\text{magn.}} + \underbrace{\frac{1}{2}\frac{\Phi}{A}\cdot\frac{\Phi}{\mu_0 A}\cdot A\cdot l}_{\text{luftgap.}} + 0$$

$$\bar{F}_m = -\bar{\nabla} W_m \Rightarrow F = -\frac{dW_m}{dl} = -\frac{1}{2}\frac{\Phi^2}{\mu_0 A}$$

Vill minsk l!

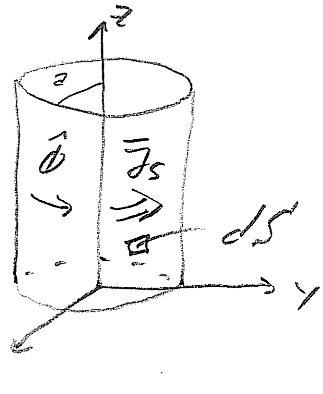
Beräkna Φ: Stoppa in ⑧ o ⑨ i ① $\Rightarrow$

$$\frac{\Phi L}{\mu_0\mu_r A} + \frac{\Phi l}{\mu_0 A} = NI \Rightarrow \Phi = \frac{\mu_0 A N I}{L/\mu_r + l} \Rightarrow$$

$$\underline{\underline{F = -\frac{\mu_0 A}{2}\left(\frac{NI}{L/\mu_r + l}\right)^2 \approx -0{,}23\ N}}$$

<u>Svar: Kraften $\frac{\mu_0 A}{2}\left(\frac{\mu_r NI}{L+\mu_r l}\right)^2 \approx 0.23\,N$ vill minska gapet</u>

3) Ytladdningstätheten som roterar kan ses som en ytströmtäthet $\bar{J}_s = \rho_s a \omega \hat{\phi}$



[Jfr $\bar{J} = nq\bar{v} = \rho \bar{v}$]

$$\bar{B}(\bar{r}) = \int_S \frac{\mu_0 \bar{J}_s \times (\bar{r} - \bar{r}')}{4\pi [\bar{r} - \bar{r}']^3} dS' \; ; \quad \bar{r} = z\hat{z}, \quad \bar{r}' = a\hat{R}' + z'\hat{z}$$

$$\underline{\underline{\bar{B}(z\hat{z})}} = \int_0^h \int_0^{2\pi} \frac{\mu_0 \rho_s a \omega \hat{\phi} \times [\overbrace{(z-z')\hat{z}}^{\hat{R} \to 0 \text{ ty } 0 \to 2\pi} \overbrace{- a\hat{R}'}^{-\hat{z}}]}{4\pi [(z'-z)^2 + a^2]^{3/2}} a \, d\phi \, dz =$$

$$= \frac{\mu_0 \rho_s a^3 \omega}{4\pi} \cdot 2\pi \int_0^h \frac{1}{[(z'-z)^2 + a^2]^{3/2}} dz \, \hat{z} = \frac{\mu_0 \rho_s a^3 \omega}{2 a^2} \left[\frac{z'-z}{\sqrt{(z'-z)^2 + a^2}} \right]_0^h \hat{z}$$

$$= \underline{\underline{\frac{\mu_0 \rho_s a \omega}{2} \left(\frac{h-z}{\sqrt{(h-z)^2 + a^2}} + \frac{z}{\sqrt{z^2 + a^2}} \right) \hat{z}}}$$

$$\underline{\underline{\text{Svar: } \bar{B}(z\hat{z}) = \frac{\mu_0 \rho_s a \omega}{2} \left(\frac{h-z}{\sqrt{(h-z)^2 + a^2}} + \frac{z}{\sqrt{z^2 + a^2}} \right) \hat{z}}}$$

4/ Börja med att ta fram det tillhörande $\bar{H}$-fältet.

$$\bar{\nabla}\times\bar{E} = -\frac{\partial \bar{B}}{\partial t} \Rightarrow \bar{\nabla}\times\bar{E} = \frac{\partial E_y}{\partial x}\hat{z} = kE_0\cos(kx-\omega t)\hat{z} = -\frac{\partial \bar{B}}{\partial t} \Rightarrow$$

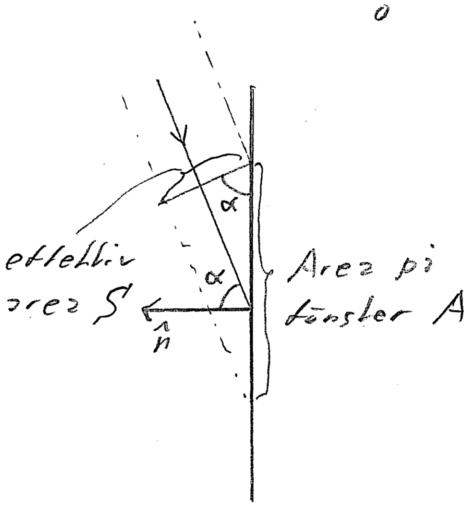
$$\bar{B} = \frac{kE_0}{\omega}\sin(kx-\omega t)\hat{z} + \underbrace{\bar{B}_0}_{\text{tidsober. konst}} \quad \text{Sätt } \bar{B}_0 = \bar{0}.$$

$$\bar{H} = \bar{B}/\mu_0 = \frac{kE_0}{\omega\mu_0}\sin(kx-\omega t)\hat{z} = \frac{E_0}{c_0\mu_0}\sin(kx-\omega t)\hat{z}$$

Poyntingvektor: $\bar{P} = \bar{E}\times\bar{H} = \dfrac{E_0^2}{c_0\mu_0}\sin^2(kx-\omega t)\hat{x}$

Tidsmedelvärde: $\langle\bar{P}\rangle = \dfrac{1}{T}\displaystyle\int_0^T \bar{P}(t)\,dt = \dfrac{\omega}{2\pi}\int_0^{2\pi/\omega}\bar{P}(t)\,dt =$

$$= \frac{\omega}{2\pi}\cdot\frac{E_0^2}{c_0\mu_0}\int_0^{2\pi/\omega}\underbrace{\frac{1}{2}\left[1-\cos 2(kx-\omega t)\right]}_{\sin^2(kx-\omega t)}dt\,\hat{x} = \frac{1}{2}\frac{E_0^2}{c_0\mu_0}\hat{x}$$



$$\langle P_{\text{fönster}}\rangle = \langle\bar{P}\rangle\cdot\hat{n} = \frac{1}{2}\frac{E_0^2}{c_0\mu_0}\cos\alpha$$

Alternativt ur figur: $S = A\cos\alpha$

Svar: Energiflödet är $\dfrac{1}{2}\dfrac{E_0^2}{c_0\mu_0}\cos\alpha \approx 0{,}66\ kW/m^2$

5) Studera området mellan plattorna $i=0$ och $i=1$.

$\bar{E} = -\bar{\nabla} V$, $\bar{D} = \varepsilon_0 \bar{E}$, $\bar{\nabla} \cdot \bar{D} = \rho = 0 \Rightarrow \nabla^2 V = 0$

Givet att $V(R, \phi, z)$ ober. av R och z. $\Rightarrow$

$$\bar{\nabla}^2 V = \frac{1}{R^2} \frac{\partial^2 V}{\partial \phi^2} = 0 \Rightarrow \frac{\partial V}{\partial \phi} = \text{konst.} = C_1 \Rightarrow V(\phi) = C_1 \phi + C_2$$

Sätt $V(0) = 0$, $V\left(\frac{\pi}{3}\right) = U \Rightarrow V(\phi) = \frac{3}{\pi} U \cdot \phi$, $\bar{E} = -\bar{\nabla} V \Rightarrow$

$$\bar{E} = \frac{-1}{R} \frac{\partial V}{\partial \phi} \hat{\phi} = \frac{-3U}{\pi R} \hat{\phi} \Rightarrow \bar{D} = \varepsilon_0 \bar{E} = \frac{-3\varepsilon_0 U}{\pi R} \hat{\phi}$$

Räkna ut laddningen på belägget med hög potential d.v.s. plattan vid $\phi = \pi/3$.

$$\rho_s = \bar{D} \cdot \hat{n} = \frac{-3\varepsilon_0 U}{\pi R} \hat{\phi} \cdot (-\hat{\phi}) = \frac{3\varepsilon_0 U}{\pi R}$$

$$Q = \int_S \rho_s \, dS = \int_0^h \int_a^b \frac{3\varepsilon_0 U}{\pi R} \, dR \, dz = \frac{3\varepsilon_0 h U}{\pi} \ln b/a$$

$$C = Q/U = \frac{3\varepsilon_0 h}{\pi} \ln b/a$$

Totalt har vi 6 parallellkopplade kondensatorer $\Rightarrow$

$$\underline{\underline{C_{tot} = 6C = \frac{18\varepsilon_0 h}{\pi} \ln b/a}}$$

<u>Svar: Totala kapacitansen är: $C_{tot} = \frac{18\varepsilon_0 h}{\pi} \ln b/a$</u>