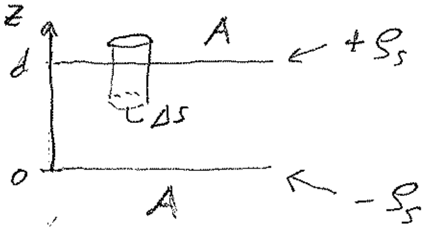
Tentamen i Elektromagnetism, TFYA13, 2010-08-18

1a)



Plattsymmetri $\Rightarrow \bar{D} = \bar{0}$ utanför och $\bar{D} = D\hat{z}$ mellan plattorna.

Gauss sats, $\oint_S \bar{D}\cdot d\bar{S} = Q_{fri,innes}$ ger:

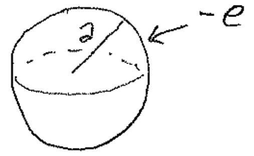
$$\oint_S \bar{D}\cdot d\bar{S} = \underbrace{D\hat{z}\cdot(-\hat{z})\Delta S}_{\text{botten}} + \underset{\substack{\text{lock}\\ \bar{D}=\bar{0}}}{0} + \underset{\substack{\text{mantel}\\ \bar{D}\perp\hat{n}}}{0} = Q_{fri,innes} = \rho_s \Delta S \Rightarrow \bar{D} = -\rho_s\hat{z}$$

$$W_e = \int_{\text{"Hela } R^3\text{"}} \frac{1}{2}\bar{E}\cdot\bar{D}\,d\tau = \frac{1}{2}\left(\frac{-\rho_s}{\varepsilon_0}\hat{z}\right)\left(-\rho_s\hat{z}\right)A\cdot d + 0 = \frac{1}{2}\frac{\rho_s^2 Ad}{\varepsilon_0}$$

Övre plattans läge ges av "d", byt till variabeln z:

$$\underline{\underline{\bar{F}_e}} = -\bar{\nabla}W_e = -\frac{d}{dz}\left(\frac{1}{2}\frac{\rho_s^2 Az}{\varepsilon_0}\right)\hat{z} = \underline{\underline{-\frac{1}{2}\frac{\rho_s^2 A}{\varepsilon_0}\hat{z}}}$$

b)



Sfäriskt symmetri: $\bar{D} = D(r)\hat{r}$
Gauss sats, $\oint_S \bar{D}\cdot d\bar{S} = Q_{fri,innes}$ } På en sfär med radie r $\Rightarrow$

$$4\pi r^2 D(r) = Q_{fri,innes} \Rightarrow \bar{D} = \frac{Q_{fri,innes}}{4\pi r^2}\hat{r} = \begin{cases} \bar{0} & ; 0 \le r < a \\ \dfrac{-e}{4\pi r^2}\hat{r} & ; a < r \end{cases}$$

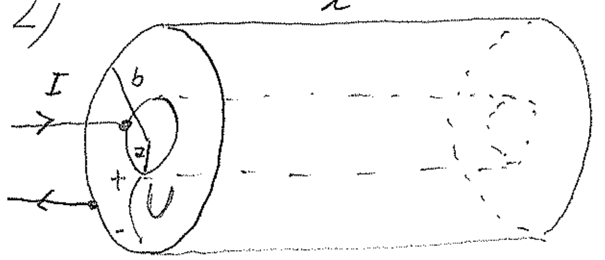
$$W_e = \int_{\text{"Hela } R^3\text{"}} \frac{1}{2}\bar{E}\cdot\bar{D}\,d\tau = \int_a^\infty\int_0^{2\pi}\int_0^{\pi} \frac{1}{2}\frac{-e}{4\pi\varepsilon_0 r^2}\cdot\frac{-e}{4\pi r^2} r^2\sin\theta\, d\theta\, d\phi\, dr =$$

$$= \frac{1}{2}\frac{e^2}{4\pi\varepsilon_0}\left[-\frac{1}{r}\right]_a^\infty = \frac{e^2}{8\pi\varepsilon_0 a} = \frac{m_e c^2}{2} \Rightarrow a = \frac{e^2}{4\pi\varepsilon_0 m_e c^2} \approx 2{,}8\cdot 10^{-15}\,\frac{m}{s}$$

Svar: a) Kraften på övre plattan är $\underline{\underline{\dfrac{\rho_s^2 A}{2\varepsilon_0}}}$

b) Elektronradien är $a = \dfrac{e^2}{4\pi\varepsilon_0 m_e c^2} \approx 2{,}8\cdot 10^{-15}\,m/s$

2)



Cylindersymmetri $\Rightarrow \bar{J} = J(R)\hat{R}$

Ansätt en ström I från inre till yttre cylindern

Strömmen genom en cylinder med radie $a < R < b$:

$$I = \int_S \bar{J} \cdot d\bar{S} = J(R) \cdot 2\pi R l \Rightarrow \bar{J} = \frac{I}{2\pi R l}\hat{R}$$

$$\bar{E} = \bar{J}/\sigma = \frac{I(\rho_0 + kR^2)}{2\pi l R}\hat{R} = \frac{I}{2\pi l}\left(\frac{\rho_0}{R} + kR\right)\hat{R}$$

$$U = \int_{Ref}^{Alt} -\bar{E} \cdot d\bar{l} = \int_b^a -\frac{I}{2\pi l}\left(\frac{\rho_0}{R} + kR\right)\hat{R}\cdot\hat{R}\, dR =$$

$$= \frac{I}{2\pi l}\left(\rho_0 \ln b/a + \frac{k}{2}[b^2 - a^2]\right) \Rightarrow$$

$$\underline{\underline{R_\Omega}} = \frac{U}{I} = \underline{\underline{\frac{1}{2\pi l}\left[\rho_0 \ln b/a + \frac{k}{2}(b^2 - a^2)\right]}}$$

b) $\rho = \bar{\nabla} \cdot \bar{D} = \bar{\nabla} \cdot \varepsilon_0\varepsilon_r \bar{E} = \frac{\varepsilon_0\varepsilon_r}{R}\frac{\partial}{\partial R}(R E_R) = \frac{\varepsilon_0\varepsilon_r}{R}\frac{\partial}{\partial R}\left(\frac{I}{2\pi l}\left[\rho_0 + kR^2\right]\right) =$

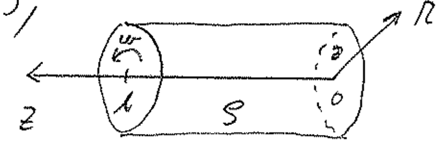
$$= \frac{\varepsilon_0\varepsilon_r k}{\pi l} I$$

$$\underline{\underline{Q_{fri,tot}}} = \int_\tau \rho\, d\tau = \int_\tau \frac{\varepsilon_0\varepsilon_r k}{\pi l} I\, d\tau = \frac{\varepsilon_0\varepsilon_r k}{\pi l} I \cdot l \cdot \pi(b^2 - a^2) = \underline{\underline{\varepsilon_0\varepsilon_r k(b^2 - a^2) I}}$$

Svar: a) Resistansen är: $R_\Omega = \frac{1}{2\pi l}\left[\rho_0 \ln\frac{b}{a} + \frac{k}{2}(b^2 - a^2)\right]$

b) Total fri laddning: $Q_{fri,tot} = \varepsilon_0\varepsilon_r k(b^2 - a^2) I$

3/



$\bar{J} \equiv nq\bar{v} = \varrho\bar{v} = \varrho R\omega\hat{\phi}$

Biot-Savarts lag $\bar{B}(\bar{r}) = \int \frac{\mu_0 \bar{J}(\bar{r}')d\tau' \times (\bar{r}-\bar{r}')}{4\pi|\bar{r}-\bar{r}'|^3}$

$$\bar{B}(z\hat{z}) = \int_0^{\ell}\int_0^{a}\int_0^{2\pi} \frac{\mu_0 \varrho R'\omega \, \underbrace{\hat{\phi}' \times (z\hat{z} - [z'\hat{z} + R'\hat{R}'])}^{\hat{R}' \Rightarrow 0,\ \text{ty } 0 \to 2\pi}}{4\pi\left([(R')^2+(z'-z)^2]\right)^{3/2}} R'd\phi' dR' dz' =$$

$$= \int_0^{\ell}\int_0^{a}\int_0^{2\pi} \frac{\mu_0 \varrho (R')^3 \omega \hat{z}}{4\pi\left((R')^2+(z'-z)^2\right)^{3/2}} d\phi' dR' dz' = \frac{\mu_0\varrho\omega}{2}\int_0^a\int_0^{\ell} \frac{(R')^3 dz' dR'}{\left((R')^2+(z'-z)^2\right)^{3/2}}\hat{z} =$$

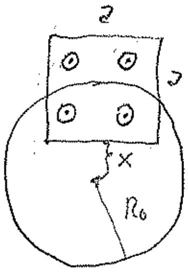
$$= \frac{\mu_0\varrho\omega}{2}\int_0^a \left[\frac{(R')^3(z'-z)}{(R')^2\sqrt{(R')^2+(z'-z)^2}}\right]_0^{\ell} dR'\hat{z} =$$

$$= \frac{\mu_0\varrho\omega}{2}\int_0^a \frac{R'(L-z)}{\sqrt{(R')^2+(L'-z)^2}} + \frac{R'z}{\sqrt{(R')^2+z^2}} \, dR'\hat{z} =$$

$$= \frac{\mu_0\varrho\omega}{2}\left\{(L-z)\left[\sqrt{(R')^2+(L-z)^2}\right]_0^a + z\left[\sqrt{(R')^2+z^2}\right]_0^a\right\}\hat{z} =$$

$$= \frac{\mu_0\varrho\omega}{2}\left\{(L-z)\left(\sqrt{a^2+(L-z)^2} - |L-z|\right) + z\left(\sqrt{a^2+z^2} - |z|\right)\right\}\hat{z}$$

Svar: $\bar{B}(z\hat{z}) = \frac{\mu_0\varrho\omega}{2}\left\{(L-z)\left(\sqrt{a^2+(L-z)^2} - |L-z|\right) + z\left(\sqrt{a^2+z^2} - |z|\right)\right\}\hat{z}$

4) 

Inför avståndet $x = v_0 t$, se figur.

Flödet genom ringen kan då skrivas:

$$\Phi = \Phi_0 - B_0 a x = \Phi_0 - B_0 a v_0 t \quad \text{där } \Phi_0 \text{ är}$$

flödet då $x=0$. $\Rightarrow \mathcal{E}_{emk} = -\frac{d\Phi}{dt} = B_0 a v_0$

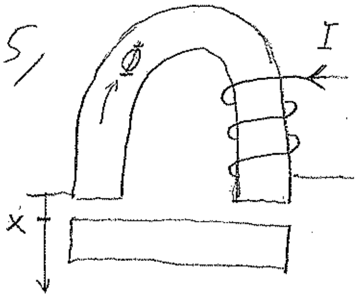
Elektriska effekten i slingan blir $P_\Omega = \mathcal{E}_{emk}^2 / R_\Omega$

Samtidigt tillförs slingan effekten mgv_0 mha gravitation. Balans $\Rightarrow$ $\mathcal{E}_{emk}^2 / R_\Omega = mgv_0 \Rightarrow$

$$\frac{B_0^2 a^2 v_0^2}{R_\Omega} = mgv_0 \Rightarrow B_0 = \frac{1}{a}\sqrt{\frac{mgR_\Omega}{v_0}} \approx 0{,}50\ \mathrm{T}$$

Svar: En magnetisk flödestäthet $B_0 = \frac{1}{a}\sqrt{\frac{mgR_\Omega}{v_0}} \approx 0{,}50\ \mathrm{T}$ ger den önskade jämviktshastigheten

---

5) 

Antag att magnetiska flödet i kretsen är $\Phi \Rightarrow \underset{\text{magnet}}{B_m} = \Phi / S_m$; $\underset{\text{ok}}{B_0} = \Phi / S_0$; $\underset{\text{papper}}{B_p} = \Phi / S_m$

$$W = \underset{\text{"Hela } R^3\text{"}}{\int} \frac{1}{2} \bar{H} \cdot \bar{B}\, d\tau = \underbrace{W_m + W_0}_{\text{ober. av } x} + 2 \cdot \frac{1}{2} \frac{\Phi}{S_m} \cdot \frac{\Phi}{\mu_0 S_m} \cdot S_m \cdot x \Rightarrow$$

$$\bar{F}_0 = -\bar{\nabla} W = -\frac{dW}{dx}\hat{x} = -\frac{\Phi^2}{\mu_0 S_m}\hat{x} \Rightarrow \Phi = \sqrt{\mu_0 S_m F_0}$$

Flödet i kretsen måste alltså hållas konstant ober. av x. Men då måste även $\bar{B}$ o $\bar{H}$ fälten i magnet och ok vara konst.

$$\oint_C \bar{H} \cdot d\bar{l} = I_{fri,oms} \Rightarrow \underbrace{H_m \cdot l_m + H_0 \cdot l_0}_{\text{Ska vara konstant } = 0} + 2 \cdot x \cdot H_l = NI$$

ty utan papper är $x=0$ o $I=0$

$$\therefore \underline{\underline{I}} = \frac{2H_l}{N} x = \frac{2\sqrt{\mu_0 S_m F_0}}{N \mu_0 S_m} x = \frac{2x}{N}\sqrt{\frac{F_0}{\mu_0 S_m}} = \underline{\underline{\frac{2}{N}\sqrt{\frac{F_0}{\mu_0 S_m}} \cdot d}}$$

Svar: Strömmen som krävs är $I = \frac{2}{N}\sqrt{\frac{F_0}{\mu_0 S_m}} \cdot d$ vilket är ca 6.3A per mm papper.