Elektromagnetism, TFYA12, 2010-05-31

1) $\bar{E} = E_0 \sin(kx - \omega t)\hat{y}$; $\bar{\nabla} \times \bar{E} = -\frac{\partial \bar{B}}{\partial t} \Rightarrow$

$$\bar{\nabla} \times \bar{E} = \frac{\partial E_y}{\partial x}\hat{z} = kE_0 \cos(kx - \omega t)\hat{z} = -\frac{\partial \bar{B}}{\partial t} \Rightarrow$$

$$\underline{\underline{\bar{B}}} = \frac{kE_0}{\omega}\sin(kx - \omega t)\hat{z} + \underset{\substack{\text{tidsoberoende}\\ \text{sätts till } \bar{0}}}{\bar{B}_0} = \underline{\underline{\frac{E_0}{c}\sin(kx - \omega t)\hat{z}}}$$

b) $\bar{\mathcal{P}} = \bar{E} \times \bar{H} = \bar{E} \times \bar{B}/\mu_0 = \frac{E_0^2}{\mu_0 c}\sin^2(kx - \omega t)\overbrace{(\hat{y} \times \hat{z})}^{\hat{x}}$

$$\langle \bar{\mathcal{P}} \rangle = \frac{1}{T}\int_0^T \bar{\mathcal{P}}(t)\,dt = \frac{E_0^2}{\mu_0 c}\frac{1}{T}\int_0^T \frac{1}{2}(1 - \cos[2(kx - \omega t)])\,dt\,\hat{x} =$$

Periodtid $= \frac{2\pi}{\omega}$

$$= \frac{E_0^2}{2\mu_0 c}\hat{x}$$

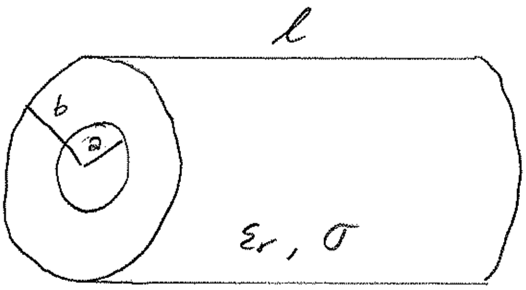
$$P_0 = \langle P \rangle = \oint_S \langle \bar{\mathcal{P}} \rangle \cdot d\bar{S} = \frac{E_0^2}{2\mu_0 c} \cdot 4\pi r^2 \Rightarrow E_0 = \pm\sqrt{\frac{\mu_0 c}{2\pi}P_0} \cdot \frac{1}{r}$$

Numeriskt: $E_0 = 3,9$ V/m

Svar: a) $\underline{\underline{\bar{B}} = \frac{E_0}{c}\sin(kx - \omega t)\hat{z}}$

b) $\underline{\underline{E_0 = \frac{1}{r}\sqrt{\frac{\mu_0 c}{2\pi}P_0} \approx 3,9 \text{ V/m}}}$

2)



Cylindersymmetri $\Rightarrow$
$\bar{D} = D(R)\hat{R}$, $\bar{J} = J(R)\hat{R}$
Inhomogent material med ledningsförmåga $\Rightarrow$ det samlas fri laddning i materialet.

Ansätt en ström I från inre cylindern mot yttre.
Strömmen måste då gå genom varje koaxiell cylinder med radie $a<R<b$. $\Rightarrow$

$$I = \int_{S'} \bar{J}\cdot d\bar{S} = \int J(R)\hat{R}\cdot\hat{R}\,dS = J(R)\int dS = J(R)\,2\pi R l \Rightarrow$$

$$\bar{J} = \frac{I}{2\pi l R}\hat{R} \Rightarrow \bar{E} = \bar{J}/\sigma = \frac{\sigma_0 + kR}{2\pi l R} I\hat{R} = \left(\frac{\sigma_0}{2\pi R} + \frac{k}{2\pi}\right)\frac{I}{l}\hat{R}$$

$$U = \int_{Ref}^{Akt} -\bar{E}\cdot d\bar{l} = \int_b^a -\left(\frac{\sigma_0}{R} + k\right)\frac{I}{2\pi l}\hat{R}\cdot\hat{R}\,dR = \left(\sigma_0 \ln\frac{b}{a} + k[b-a]\right)\frac{I}{2\pi l}$$

$$\underline{\underline{R_\Omega}} = U/I = \underline{\underline{\frac{1}{2\pi l}\left(\sigma_0 \ln b/a + k[b-a]\right)}}$$

b) $\bar{D} = \varepsilon_0\varepsilon_r\bar{E}$; $\bar{\nabla}\cdot\bar{D} = \rho \Rightarrow \rho_f = \varepsilon_0\varepsilon_r\bar{\nabla}\cdot\bar{E} = \varepsilon_0\varepsilon_r\frac{1}{R}\frac{\partial}{\partial R}(RE_R) =$

$$= \frac{\varepsilon_0\varepsilon_r}{R}\frac{kI}{2\pi l}$$ Total fri laddn. $\underline{\underline{Q_{tot}}} = \int_\tau \rho\, d\tau =$

$$= \int_0^l\int_a^b\int_0^{2\pi} \frac{\varepsilon_0\varepsilon_r kI}{2\pi l R} R\,d\phi\,dR\,dz = \frac{\varepsilon_0\varepsilon_r kI}{2\pi l}\cdot 2\pi\cdot l\cdot(b-a) =$$

$$\underline{\underline{= \varepsilon_0\varepsilon_r kI(b-a)}}$$

Svar: a) Resistansen är: $\frac{1}{2\pi l}\left(\sigma_0 \ln b/a + k[b-a]\right)$

b) Total fri laddning: $\varepsilon_0\varepsilon_r kI(b-a)$

3) Cylindersymmetri $\Rightarrow \bar{H} = H(R)\hat{\phi}$
Cirkulationssatsen $\oint \bar{H}\cdot d\bar{l} = I_{fri.oms.}$ $\Bigg\} \Rightarrow 2\pi R H(R) = I_{fri.oms}$

$$\Rightarrow \bar{H} = \frac{I_{fri.oms}}{2\pi R}\hat{\phi} = \begin{Bmatrix} I_{fri.oms} = \frac{R^2}{a^2} I_0 & 0 \le R \le a \\ I_{fri.oms} = I_0 & a \le R \end{Bmatrix} =$$

$$= \begin{cases} \frac{R I_0}{2\pi a^2}\hat{\phi} & 0 \le R \le a \\ \frac{I_0}{2\pi R}\hat{\phi} & a \le R \end{cases}$$

a) I xz-planet vid slingan är: $\bar{H} = \frac{I_0}{2\pi X}\hat{y} \Rightarrow$

$$\bar{B} = \mu_0 \bar{H} = \frac{\mu_0 I_0}{2\pi X}\hat{y} \Rightarrow \Phi = \int_S \bar{B}\cdot d\bar{S} = \int_0^c \int_d^{d+b} \frac{\mu_0 I_0}{2\pi X}\hat{y}\cdot\hat{y}\, dx\, dz \Rightarrow$$

$$\underline{\underline{\Phi}} = \frac{\mu_0 I_0 c}{2\pi} \ln\left(\frac{d+b}{d}\right) = \underline{\underline{\frac{\mu_0 c I_0}{2\pi} \ln(1 + b/d)}}$$

b) $\bar{B} = \mu_0(\bar{H} + \bar{M}) \Rightarrow \bar{M} = \bar{B}/\mu_0 - \bar{H} = \mu_r \bar{H} - \bar{H} = (\mu_r - 1)\bar{H} \Rightarrow$

$$\bar{M} = \frac{(\mu_r - 1) R I_0}{2\pi a^2}\hat{\phi}$$

$$\underline{\underline{\bar{J}_m}} = \bar{\nabla}\times\bar{M} = \frac{1}{R}\frac{\partial}{\partial R}(R M_\phi)\hat{z} = \underline{\underline{\frac{(\mu_r - 1) I_0}{\pi a^2}\hat{z}}}$$

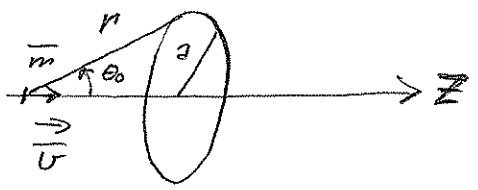
c) $\underline{\underline{\bar{J}_{sm}}} = \bar{M}\times\hat{n} = \frac{(\mu_r - 1) a I_0}{2\pi a^2}\hat{\phi}\times\hat{R} = \underline{\underline{-\frac{(\mu_r - 1) I_0}{2\pi a}\hat{z}}}$

Svar: a) $\underline{\underline{\Phi = \frac{\mu_0 c I_0}{2\pi} \ln(1 + b/d)}}$

b) $\underline{\underline{\bar{J}_m = \frac{(\mu_r - 1) I_0}{\pi a^2}\hat{z}}}$

c) $\underline{\underline{\bar{J}_{sm} = -\frac{(\mu_r - 1) I_0}{2\pi a}\hat{z}}}$

4)



Använd först ett störiskt koordinatsystem med centrum av dipolen och $\hat{z}$ längs den givna $\hat{z}$.

Magnetiska flödet Φ genom slingan är då:

$$\Phi = \int_S \bar{B}\cdot d\bar{S} = \int_0^{\theta_0}\int_0^{2\pi} \frac{\mu_0 m}{4\pi r^3}(2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta})\cdot\hat{r}\, r^2 \sin\theta\, d\theta\, d\phi =$$

$$= \frac{\mu_0 m}{4\pi r}\cdot 2\pi \sin^2\theta_0 = \left\{\sin\theta_0 = \frac{a}{r}\right\} = \frac{\mu_0 m a^2}{2r^3}$$

där $r = \sqrt{a^2+z^2}$ och $z = vt$ ($t=0$ när dipolen passerar genom slingan)

$$\mathcal{E}_{emk} = -\frac{d\Phi}{dt} = -\frac{d\Phi}{dr}\frac{dr}{dz}\frac{dz}{dt} = \frac{3\mu_0 m a^2}{2r^4}\cdot\frac{z}{r}\cdot v = \frac{3\mu_0 m a^2 z v}{2r^5} \quad (*)$$

Som är maximal då $0 = \dfrac{d\mathcal{E}_{emk}}{dz} = \dfrac{3\mu_0 m a^2 v}{2}\left(\dfrac{1}{r^5} - \dfrac{5z}{r^6}\cdot\dfrac{z}{r}\right) \Rightarrow$

$\Rightarrow r^2 = 5z^2 \Rightarrow a^2+z^2 = 5z^2 \Rightarrow z^2 = a^2/4 \Rightarrow z_{max} = \pm a/2$

och $r_{max} = \sqrt{a^2 + a^2/4} = \dfrac{\sqrt{5}}{2}a$ ger i $(*)$

$$\underline{\underline{\mathcal{E}_{emk}}} = \frac{(\pm)\,3\mu_0 m a^2\cdot a v}{2\cdot\left(\frac{\sqrt{5}}{2}a\right)^5\cdot 2} = \underline{\underline{\frac{24\mu_0 m v}{25\sqrt{5}\,a^2}}}$$

Svar: Den maximala inducerade elektromotoriska kraften är: $\dfrac{24\mu_0 m v}{25\sqrt{5}\,a^2}$

5) Ansätt en laddning Q på bubblan.

Sfärisk symmetri $\Rightarrow \bar{D} = D(r)\hat{r}$

Gauss sats $\oint_S \bar{D}\cdot d\bar{S} = Q_{fri,innes}$

$\Rightarrow \bar{D} = \frac{Q_{fri,innes}}{4\pi r^2}\hat{r}$

$$\Rightarrow \bar{E} = \frac{Q_{fri,innes}}{4\pi\varepsilon_0 r^2}\hat{r} = \begin{cases} \bar{0} & 0 \le r < a \\ \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r} & a < r \end{cases}$$

$$W_e = \int_{\text{"Hela } R^3\text{"}} \frac{1}{2}\bar{E}\cdot\bar{D}\,d\tau = \int_a^\infty\int_0^{2\pi}\int_0^{\pi} \frac{1}{2\varepsilon_0}\left(\frac{Q}{4\pi r^2}\right)^2 r^2 \sin\theta\, d\theta\, d\phi\, dr =$$

$$= \frac{1}{2\varepsilon_0}\left(\frac{Q}{4\pi}\right)^2 \cdot 4\pi\left[\frac{-1}{r}\right]_a^\infty = \frac{Q^2}{8\pi\varepsilon_0 a}$$

Elektrostatiska kraften som vill öka bubblans radie blir:

$$\bar{F}_e = -\frac{\partial W_e}{\partial a}\hat{r} = \frac{Q^2}{8\pi\varepsilon_0 a^2}\hat{r} \Rightarrow p = \frac{|\bar{F}_e|}{A} = \frac{|\bar{F}_e|}{4\pi a^2} = \frac{1}{2\varepsilon_0}\left(\frac{Q}{4\pi a^2}\right)^2$$

Q fås från att $E_{max} = \frac{Q}{4\pi\varepsilon_0 a^2} \Rightarrow Q = 4\pi\varepsilon_0 a^2 E_{max}$

$$\therefore\; p = \frac{\varepsilon_0}{2}E_{max}^2 \approx 17{,}7 \text{ Pa}$$

Svar: Det elektrostatiska trycket $p = \frac{\varepsilon_0}{2}E_{max}^2 \approx 17{,}7$ Pa vill öka radien på såpbubblan.