entamen i Elektromagnetism, TFYA13/NFYB03 3/6-09

1) a) Använd $\bar{\nabla} \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$

$$\bar{\nabla} \times \bar{E} = \frac{\partial E_y}{\partial x}\hat{z} = kE_0 \cos(kx-\omega t)\hat{z} = -\frac{\partial \bar{B}}{\partial t} \Rightarrow$$

$$\bar{B} = \frac{k}{\omega}E_0 \sin(kx-\omega t)\hat{z} + \bar{B}_0$$ Tidsober. (Kan sättas till $\bar{0}$)

$$\bar{B} = \mu_0 \bar{H} \Rightarrow \underline{\underline{\bar{H}}} = \frac{\bar{B}}{\mu_0} = \underline{\underline{\frac{k}{\omega}\frac{E_0}{\mu_0}\sin(kx-\omega t)\hat{z}}}$$

b) $$\bar{P} = \bar{E} \times \bar{H} = \frac{k}{\omega}\frac{1}{\mu_0}E_0^2 \sin^2(kx-\omega t)\underbrace{(\hat{y}\times\hat{z})}_{\hat{x}}$$

$$\langle \bar{P} \rangle = \frac{1}{T}\int_0^T \bar{P}(t)\,dt = \frac{k}{\omega}\frac{1}{\mu_0}E_0^2 \frac{1}{T}\int_0^T \sin^2(kx-\omega t)\,dt\,\hat{x} =$$

En period.

$$= \frac{k}{\omega}\frac{1}{\mu_0}E_0^2\frac{1}{T}\int_0^T \frac{1}{2}\left(1-\cos[2(kx-\omega t)]\right)dt\,\hat{x} = \frac{k}{\omega}\frac{1}{\mu_0}E_0^2\frac{1}{2}\hat{x}$$

Svar: $\underline{\underline{\bar{H}}} = \underline{\underline{\frac{k}{\omega}\frac{E_0}{\mu_0}\sin(kx-\omega t)\hat{z}}}$

$$\underline{\underline{\langle \bar{P} \rangle = \frac{1}{c_0}\frac{1}{\mu_0}E_0^2\frac{1}{2}\hat{x} \approx 1{,}3\cdot 10^3\,\hat{x} \ \ W/m^2}}$$

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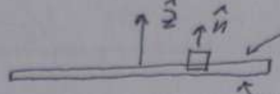
2) $$V(\bar{r}) = \int_{\circledast} \frac{dQ'}{4\pi\varepsilon_0 |\bar{r}-\bar{r}'|}$$ Beräkna potentialen i

skivans mittpunkt $\Rightarrow \bar{r} = \bar{0},\ \bar{r}' = R'\hat{R}' \Rightarrow$

$$V_0 = V(0) = \int_0^a\int_0^{2\pi} \frac{2\cdot\alpha \quad R'\,d\phi\,dR'}{4\pi\varepsilon_0\sqrt{a^2-R'^2}\cdot R'} = \frac{\alpha}{\varepsilon_0}\left[\arcsin\left(\frac{R'}{a}\right)\right]_0^a = \frac{\alpha}{\varepsilon_0}\frac{\pi}{2} \Rightarrow$$

$$\underline{\underline{\alpha = 2\varepsilon_0 V_0/\pi}}$$

2b,



Använd liten Gauss låda och $\oint_S \bar{D}\cdot d\bar{S} = Q_{fri,inner}$ $\Big\}\Rightarrow$

$$\underbrace{\hat{n}\cdot\bar{D}(R)\Delta S}_{lock} + \underbrace{0}_{\substack{\hat{n}\perp\bar{D} \\ mantell}} + \underbrace{0}_{\substack{Botten\ i \\ metall}} = \varrho_s \Delta S \Rightarrow D(R) = \varrho_s \ , \ \bar{D} = \varepsilon_0 \bar{E}$$

$$\Rightarrow \bar{E} = \varrho_s/\varepsilon_0\,\hat{z} = \frac{\alpha}{\varepsilon_0\sqrt{a^2-R^2}}\hat{z} = \frac{2V_0}{\pi\sqrt{a^2-R^2}}\hat{z}$$

Svar: a) $\alpha = 2\varepsilon_0 V_0/\pi$

b) $\bar{E}(R) = \dfrac{\alpha}{\varepsilon_0\sqrt{a^2-R^2}}\hat{z} = \dfrac{2V_0}{\pi\sqrt{a^2-R^2}}\hat{z}$

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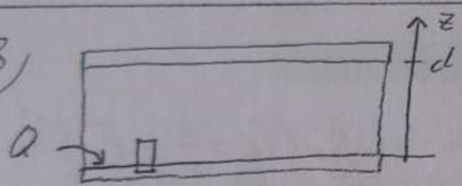
3)



Plattsymmetri $\Rightarrow \bar{D} = D(z)\hat{z}$
Ansätt laddn. Q på undre plattans översida.

Använd Gaussburk och $\oint_S \bar{D}\cdot d\bar{S} = Q_{fri,inner}$ $\Big\}\Rightarrow \hat{n}\cdot D(z)\hat{z}\Delta S + \underbrace{0}_{\bar{D}\perp\hat{n}} + \underbrace{0}_{\bar{D}=\bar{0}} = \dfrac{Q}{A}\Delta S$

$$\Rightarrow \bar{D} = \frac{Q}{A}\hat{z} = \varepsilon_0\bar{E} + \bar{P} = \varepsilon_0\bar{E} + \alpha\bar{E} + P_0\hat{z} \Rightarrow$$

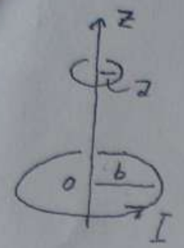
$$(\varepsilon_0+\alpha)\bar{E} = \left(\frac{Q}{A} - P_0\right)\hat{z} \Rightarrow \bar{E} = \frac{Q/A - P_0}{\varepsilon_0+\alpha}\hat{z}$$

$$U = \int_{Ref}^{Att} -\bar{E}\cdot d\bar{l} = \int_d^0 -\frac{Q/A - P_0}{\varepsilon_0+\alpha}\hat{z}\cdot\hat{z}\,dz = \frac{Q/A - P_0}{\varepsilon_0+\alpha}\cdot d \Rightarrow$$

$$\frac{(\varepsilon_0+\alpha)U}{d} = \frac{Q}{A} - P_0 \Rightarrow Q = \frac{A(\varepsilon_0+\alpha)}{d}U + P_0 A \Rightarrow$$

$$C(U) = Q/U = \frac{A(\varepsilon_0+\alpha)}{d} + P_0 A/U$$

Svar: $C(U) = \dfrac{A(\varepsilon_0+\alpha)}{d} + \dfrac{P_0 A}{U}$



Fället längs z-axeln orsakat av den stora slingan fås från Biot-Savarts

$$\bar{B}(z\hat{z}) = \int_0^{2\pi} \frac{\mu_0 I \, d\bar{l}' \times (\bar{r}-\bar{r}')}{4\pi |\bar{r}-\bar{r}'|^3} \; ; \; \begin{cases} \bar{r} = z\hat{z} \\ \bar{r}' = b\hat{R}' \\ d\bar{l}' = b\, d\phi' \hat{\phi}' \end{cases}$$

$$d\bar{l}' \times (\bar{r}-\bar{r}') = b\, d\phi' \overbrace{\underbrace{\hat{\phi}' \times (z\hat{z}}_{\hat{R}'} - b\hat{R}')}^{-\hat{z}} =$$

$$= b(z\hat{R}' + b\hat{z})\, d\phi'$$

$\Rightarrow \bar{0}$ ty $0 \to 2\pi$

$$\bar{B}(z\hat{z}) = \int_0^{2\pi} \frac{\mu_0 I b^2 \, d\phi'}{4\pi (z^2+b^2)^{3/2}} \hat{z} = \frac{\mu_0 I b^2}{2(z^2+b^2)^{3/2}} \hat{z}$$

För lilla slingan antar vi att $\bar{B}$ i centrum gäller aproximativt över hela.

$\Rightarrow$ Flödet genom lilla slingan blir:

$$\Phi = \pi a^2 \cdot \frac{\mu_0 I b^2}{2(z^2+b^2)^{3/2}} \equiv \frac{A}{(z^2+b^2)^{3/2}} \quad \text{sätt } z = vt$$

$$\mathcal{E} = -\frac{d\Phi}{dt} = -\frac{d\Phi}{dz} \cdot \frac{dz}{dt} = \frac{3}{2} \frac{\cdot 2zA}{(z^2+b^2)^{5/2}} \cdot v = 3Av \frac{z}{(z^2+b^2)^{5/2}}$$

Max då $\frac{d}{dz}\left(\frac{z}{(z^2+b^2)^{5/2}}\right) = \frac{1}{(z^2+b^2)^{5/2}} - \frac{5}{2} \cdot \frac{2 \cdot z^2}{(z^2+b^2)^{7/2}} = 0$

$$\Rightarrow z^2 + b^2 - 5z^2 = 0 \Rightarrow z^2 = b^2/4 \Rightarrow z = b/2$$

$$\mathcal{E}_{max} = 3Av \frac{b/2}{(b^2/4 + b^2)^{5/2}} = 3 \cdot \frac{\pi a^2 \mu_0 I b^2}{2} \cdot v \cdot \frac{b \cdot 2^5}{2 \cdot b^5 \cdot 5^2 \cdot \sqrt{5}} =$$

$$= \frac{24}{25} \frac{\pi}{\sqrt{5}} \frac{\mu_0 I a^2 v}{b^2}$$

Svar: $\mathcal{E}_{max} = \frac{24}{25} \frac{\pi}{\sqrt{5}} \cdot \frac{\mu_0 I a^2 v}{b^2}$

5) Cirkulationssatsen: $\oint_C \bar{H}\cdot d\bar{l} = I_{omsluten} \Rightarrow$

$H_1 l_1 + H_2 l_2 + 2t H_3 = 0$ där fältet i papperet hallats för H_3 (lika i båda pga symmetri).

$\bar{\nabla}\cdot\bar{B} = \bar{0} \Rightarrow \oint_S \bar{B}\cdot d\bar{S} = 0 \Rightarrow \Phi_1 = \Phi_2 = \Phi_3$ men

$S_1 = S_2 = S_3 \Rightarrow B_1 = B_2 = B_3$

Materialsamband: $B_3 = \mu_0 H_3$, $B_2 = \mu_0 \mu_r H_2 \Rightarrow$

$H_1 l_1 + \frac{l_2}{\mu_0\mu_r} B_1 + \frac{2t}{\mu_0} B_1 = 0 \Rightarrow H_1 = -\left(\frac{l_2}{\mu_r} + 2t\right)\frac{1}{\mu_0 l_1} B_1$

Ritas denna linje in i figuren ger skärningen med kurvan att $B_1 = 0{,}9\,T$

Magnetisk energi: $W_m = \int_{\text{"Hela } R^3\text{"}} \frac{1}{2}\bar{B}\cdot\bar{H}\, d\tau = W_{ok} + W_{magn} + \frac{2}{2} B_3 H_3 S_3 t =$

$= W_{ok} + W_{magn} + \frac{B_1^2}{\mu_0} S_1 t$ (Då papper har samma magn. egenskaper som luft behöver vi inte införa något luftgap.)

$\bar{F}_m = -\bar{\nabla} W_m \Rightarrow F_m = -\frac{\partial W_m}{\partial t} = 0 + 0 - \frac{B_1^2}{\mu_0} S_1 \approx -129\,N$

Vill minska t! ok!

Svar: Papperet trycks ihop med kraften 129N.