Tentamen i Elektromagnetism, TFYA13, 2007-08-13

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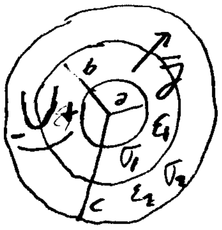


Cylinder. Ström I från inre till yttre; Symmetri $\Rightarrow \bar{J} = J(R)\hat{R}$; $I = \int \bar{J}\cdot d\bar{S}$, cylinder med radie R $\Rightarrow$

$$I = \int J(R)\hat{R}\cdot\hat{R}\,dS = 2\pi R l\, J(R) \Rightarrow \bar{J} = \frac{I}{2\pi R l}\hat{R}\,;\quad a<R<c$$

$$\bar{J} = \sigma\bar{E} \Rightarrow \bar{E} = \bar{J}/\sigma,\quad U = \int_{Ref}^{Akt} -\bar{E}\cdot d\bar{l} = \int_c^a -\bar{E}\cdot d\bar{l} \Rightarrow$$

$$U = \int_c^b -\frac{I}{2\pi l\sigma_2 R}\hat{R}\cdot\hat{R}\,dR + \int_b^a -\frac{I}{2\pi l\sigma_1 R}\hat{R}\cdot\hat{R}\,dR =$$

$$= \frac{I}{2\pi l}\left\{\frac{1}{\sigma_2}\ln c/b + \frac{1}{\sigma_1}\ln b/a\right\} \Rightarrow \underline{\underline{R_\Omega}} = \frac{U}{I} = \underline{\underline{\frac{1}{2\pi l}\left\{\frac{1}{\sigma_2}\ln\frac{c}{b} + \frac{1}{\sigma_1}\ln\frac{b}{a}\right\}}}$$

b, $\bar{D} = \varepsilon_0\varepsilon_r\bar{E} \Rightarrow \bar{D}_2 = \varepsilon_0\varepsilon_2\bar{E}_2 = \dfrac{\varepsilon_0\varepsilon_2 I}{2\pi l\sigma_2 R}\hat{R}$

$$\bar{D}_1 = \ldots = \frac{\varepsilon_0\varepsilon_1 I}{2\pi l\sigma_1 R}\hat{R}$$

Använd Gauss sats på cylinder med innerradie b_- och ytterradie b_+ dvs precis innanför resp. utanför gränsytan.

$$\oint_S \bar{D}\cdot d\bar{S} = Q_{fri,innes} \Rightarrow \int_{R=b_-} \bar{D}_1\cdot -\hat{R}\,dS + \int_{R=b_+}\bar{D}_2\cdot\hat{R}\,dS + \underset{\substack{\uparrow\\ \text{cirkelringar}}}{0} =$$

$$= \frac{-\varepsilon_0\varepsilon_1 I}{2\pi l\sigma_1 b_-}\cdot 2\pi b_- l + \frac{\varepsilon_0\varepsilon_2 I}{2\pi l\sigma_2 b_+}2\pi b_+ l = \underline{\underline{\varepsilon_0\left(\frac{\varepsilon_2}{\sigma_2} - \frac{\varepsilon_1}{\sigma_1}\right)I}}$$

<u>Svar a, Resistansen är: $\dfrac{1}{2\pi l}\left\{\dfrac{1}{\sigma_2}\ln\dfrac{c}{b} + \dfrac{1}{\sigma_1}\ln\dfrac{b}{a}\right\}$</u>

<u>b, Total fri laddning: $\varepsilon_0\left(\dfrac{\varepsilon_2}{\sigma_2} - \dfrac{\varepsilon_1}{\sigma_1}\right)I$</u>

2,

$\frac{a}{\rho_0}$

$$\left.\begin{array}{l}\text{Stor sfäris symmetri} \Rightarrow \bar{D} = D(r)\hat{r} \\ \text{Gauss sats } \oint_S \bar{D}\cdot d\bar{S} = Q_{fri,innes}\end{array}\right\} \Rightarrow$$

$$\left.\begin{array}{l}\oint_S \bar{D}\cdot d\bar{S} \underset{\substack{\uparrow \\ \text{Stor}}}{=} 4\pi r^2 D(r) = Q_{fri,innes} \quad ; \quad r \geq 0 \\ Q_{fri,innes} = \int_\tau \rho_0 \, d\tau = \begin{cases} \frac{4\pi r^3}{3}\rho_0 & ; \ 0 \leq r \leq a \\ \frac{4\pi a^3}{3}\rho_0 & ; \ a \leq r \end{cases}\end{array}\right\} \Rightarrow$$

$$\bar{D} = \begin{cases} \frac{r}{3}\rho_0 \hat{r} & ; \ 0 \leq r \leq a \\ \frac{a^3}{3r^2}\rho_0 \hat{r} & ; \ a \leq r \end{cases}$$

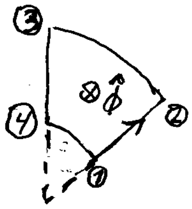
$$W_e = \underset{\text{"Hela } R^3\text{"}}{\int \frac{1}{2}\bar{E}\cdot\bar{D}\,d\tau} = \underset{\text{"Hela } R^3\text{"}}{\int \frac{1}{2}\frac{\bar{D}}{\varepsilon_0}\cdot\bar{D}\,d\tau} = \int_0^a\int_0^{2\pi}\int_0^{\pi} \frac{1}{2\varepsilon_0}\left(\frac{r\rho_0}{3}\right)^2 r^2\sin\theta\,d\theta\,d\phi\,dr +$$

$$+\int_a^\infty\int_0^{2\pi}\int_0^{\pi} \frac{1}{2\varepsilon_0}\left(\frac{a^3}{3r^2}\rho_0\right)^2 r^2\sin\theta\,d\theta\,d\phi\,dr = \frac{\rho_0^2}{18\varepsilon_0}4\pi\int_0^a r^4\,dr + \frac{\rho_0^2 a^6}{18\varepsilon_0}4\pi\int_a^\infty \frac{1}{r^2}\,dr =$$

$$= \frac{2\pi\rho_0^2}{9\varepsilon_0}\left\{\frac{a^5}{5} + \frac{5}{5}\frac{a^6}{a}\right\} = \frac{4\pi\rho_0^2 a^5}{15\varepsilon_0}$$

Svar: Det sökta arbetet är $\underline{\underline{\frac{4\pi\rho_0^2 a^5}{15\varepsilon_0}}}$

3,



Vi börjar med att använda $\mathcal{E} = \oint_C \bar{E}\cdot d\bar{l}$

$$\mathcal{E} = \int_{①}^{②} \underbrace{\bar{E}\cdot\hat{r}\,dr}_{0 \text{ ty } \bar{E}=E\hat{\theta}} + \int_{②}^{③} \bar{E}\cdot\hat{\theta}\,b\,d\theta + \int_{③}^{④} \underbrace{\bar{E}\cdot\hat{r}\,dr}_{0 \text{ ty } \bar{E}=E\hat{\theta}} + \int_{④}^{①} \bar{E}\cdot\hat{\theta}\,a\,d\theta =$$

$$= 0 + \int_{\theta_0}^{0} \frac{\alpha}{b}\sin\theta\sin(\omega t - kb)\,\hat{\theta}\cdot\hat{\theta}\,b\,d\theta + 0 +$$

$$+ \int_{0}^{\theta_0} \frac{\alpha}{a}\sin\theta\sin(\omega t - ka)\,\hat{\theta}\cdot\hat{\theta}\,a\,d\theta =$$

$$= \alpha\sin(\omega t - kb)\left[-\cos\theta\right]_{\theta_0}^{0} + \alpha\sin(\omega t - ka)\left[-\cos\theta\right]_{0}^{\theta_0} =$$

$$= \underline{\underline{\alpha(1-\cos\theta_0)\left(\sin(\omega t - ka) - \sin(\omega t - kb)\right)}}$$

$$\frac{\partial\bar{B}}{\partial t} = \frac{\alpha}{r}\cdot\frac{k}{\omega}\cdot\omega\cdot\sin\theta\cos(\omega t - kr)\,\hat{\phi}$$

$$\mathcal{E} = \int_S -\frac{\partial\bar{B}}{\partial t}\cdot d\bar{S} = \int_a^b\int_0^{\theta_0} -\frac{\alpha}{\cancel{r}}\frac{k}{\cancel{\omega}}\cdot\cancel{\omega}\cdot\sin\theta\cos(\omega t - kr)\,\underbrace{\hat{\phi}\cdot(-\hat{\phi})}_{1}\,r\,d\theta\,dr =$$

$$= \int_a^b \alpha k\left[-\cos\theta\right]_0^{\theta_0}\cos(\omega t - kr)\,dr = \int_a^b \alpha k(1-\cos\theta_0)\cos(\omega t - kr)\,dr =$$

$$= \alpha\cancel{k}(1-\cos\theta_0)\left[-\frac{\sin(\omega t - kr)}{\cancel{k}}\right]_a^b =$$

$$= \alpha(1-\cos\theta_0)\left[-\sin(\omega t - kb) + \sin(\omega t - ka)\right] =$$

$$= \underline{\underline{\alpha(1-\cos\theta_0)\left[\sin(\omega t - ka) - \sin(\omega t - kb)\right]}}$$

$$\underline{\underline{\text{Svar: } \mathcal{E} = \oint_C \bar{E}\cdot d\bar{l} = \int_S -\frac{\partial\bar{B}}{\partial t}\cdot d\bar{S} =}}$$

$$= \underline{\underline{\alpha(1-\cos\theta_0)\left[\sin(\omega t - ka) - \sin(\omega t - kb)\right]}}$$

4/ a) $\bar{B} = \bar{\nabla} \times \bar{A} = -\frac{\partial A_z}{\partial R}\hat{\phi} = \alpha_2 \mu_0 \frac{2}{R}\hat{\phi}$, $R > a$ $\Rightarrow$

$\bar{B}(R=b) = \frac{2\alpha_2\mu_0}{b}\hat{\phi}$

$\underline{\underline{\bar{F}_m}} = q\bar{v} \times \bar{B} = qv\hat{z} \times \frac{2\alpha_2\mu_0}{b}\hat{\phi} = \underline{\underline{\frac{2\alpha_2\mu_0 qv}{b}(-\hat{R})}}$

b) $\bar{\nabla} \times \bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t}$; $\bar{B} = \mu_0 \bar{H}$; $\bar{B} = \bar{\nabla} \times \bar{A}$; tids ober.

$\bar{B}_2 = \bar{B}(R > a) = \frac{2\alpha_2\mu_0}{R}\hat{\phi}$ enligt ovan.

$\bar{\nabla} \times \frac{\bar{B}}{\mu_0} = \bar{J} + \bar{0} \Rightarrow \bar{J} = \frac{1}{\mu_0}\frac{1}{R}\left(\frac{\partial}{\partial R} R B_\phi\right)\hat{z} = \frac{1}{\mu_0}\frac{1}{R}\left(\frac{\partial}{\partial R} R \frac{2\alpha_2\mu_0}{R}\right)\hat{z} = \bar{0}$

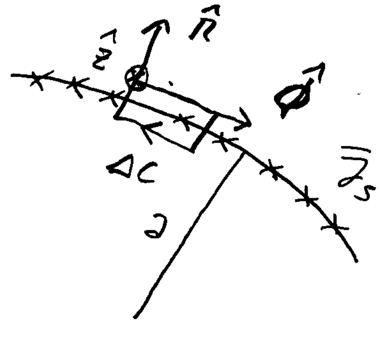
$\therefore$ $\underline{\underline{\bar{J} = \bar{0} \quad R > a}}$

$\bar{B}_1 = \bar{B}(0 \le R < a) = \bar{\nabla} \times \bar{A} = -\frac{\partial A_z}{\partial R}\hat{\phi} = \alpha_1 \mu_0 \frac{2R}{a^2}\hat{\phi}$; $0 \le R < a$

$\bar{\nabla} \times \frac{\bar{B}}{\mu_0} = \bar{J} + \bar{0} \Rightarrow \underline{\underline{\bar{J}}} = \frac{1}{\mu_0}\frac{1}{R}\left(\frac{\partial}{\partial R} R B_\phi\right)\hat{z} = \frac{1}{\mu_0 R}\left(\frac{\partial}{\partial R} R \frac{\alpha_1\mu_0 2R}{a^2}\right)\hat{z} =$

$\underline{\underline{= \frac{4\alpha_1}{a^2}\hat{z} \quad 0 \le R < a}}$

Men vi måste även undersöka randen, dvs $R = a$ vilket görs mha Stokes sats



$\oint_C \bar{H} \cdot d\bar{l} = I_{fri\,omsl} \Rightarrow$

$\Delta C \cdot H_\phi(a_+) - \Delta C\, H_\phi(a_-) =$

$= \Delta C\left[\frac{2\alpha_2}{a} - \frac{2\alpha_1}{a}\right] = \Delta C \cdot J_s \Rightarrow$

$\bar{J}_s = \frac{2}{a}(\alpha_2 - \alpha_1)\hat{z}$

Svar: Kraften blir: $\bar{F}_m = \frac{2\alpha_2\mu_0 qv}{b}(-\hat{R})$

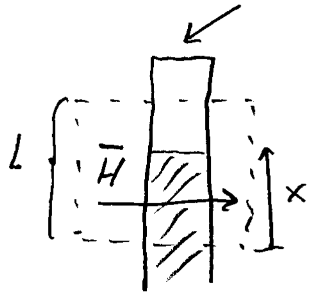
och orsakas av strömtätheten

$\bar{J} = 4\alpha_1/a^2\,\hat{z}$; $0 \le R < a$ och ytströmtätheten

$\bar{J}_s = \frac{2}{a}(\alpha_2 - \alpha_1)\hat{z}$ vid $R = a$.

5, Studera magnetiska energin

Ansätt tvärsnittsarea A



$$W_m = \int \tfrac{1}{2}\bar{B}\cdot\bar{H}\,d\tau = \int \tfrac{1}{2}\bar{B}\cdot\bar{H}\,d\tau + \int \tfrac{1}{2}\bar{B}\cdot\bar{H}\,d\tau +$$

"Hela R^3" — Vätskan i fältet — Luften i röret i fältet över vätskan

$$+ \int \tfrac{1}{2}\bar{B}\cdot\bar{H}\,d\tau =$$

Allt annat

$$= \tfrac{1}{2}\mu_0\mu_r H\cdot H\cdot A\cdot x + \tfrac{1}{2}\mu_0 H\cdot H\cdot A\cdot(L-x) + W_{rest} =$$

$$= \tfrac{1}{2}\mu_0(\mu_r-1)H^2 A x + \tfrac{1}{2}\mu_0 H^2 A L + W_{rest}$$

$$\bar{F}_m = + \bar{\nabla} W_m = + \frac{\partial W_m}{\partial x}\hat{x} = \tfrac{1}{2}\mu_0(\mu_r-1)H^2 A = \tfrac{1}{2}\mu_0 \chi_m H^2 A$$

Räkna med "+" ty H=konst vilket ger att flödet ändras när vätskeytan rör sig i höjdled.

Kraftbalans: $\tfrac{1}{2}\mu_0\chi_m H^2 A\hat{x} - g\cdot\rho_m\cdot A\cdot h\hat{x} = \bar{0} \Rightarrow$

$$\underline{\underline{\chi_m = \frac{2g\rho_m}{\mu_0 H^2}h}}$$

Svar: Vätskans magnetiska susceptibilitet är $\chi_m = \dfrac{2g\rho_m}{\mu_0 H^2}h$