Tentamen i Elektromagnetism, TFYA13, 2007-06-02

1/a) Använd $\bar{\nabla}\times\bar{E} = -\frac{\partial \bar{B}}{\partial t}$ på $\bar{E} = E_0 \sin(kx-\omega t)\hat{y}$

$$\bar{\nabla}\times\bar{E} = \frac{\partial E_y}{\partial x}\hat{z} = kE_0\cos(kx-\omega t)\hat{z} = -\frac{\partial \bar{B}}{\partial t}$$

Integrera m.a.p. tiden $\Rightarrow$

$$\underline{\bar{B} = \frac{kE_0}{\omega}\sin(kx-\omega t)\,\hat{z}} + \bar{B}_0$$ tidsober. sätt till $\bar{0}$.

b) Börja med att beräkna Poynting vektorn

$$\bar{P} = \bar{E}\times\bar{H} = \bar{E}\times\bar{B}/\mu_0 = E_0\sin(kx-\omega t)\hat{y}\times\frac{kE_0}{\omega\mu_0}\sin(kx-\omega t)\hat{z} =$$

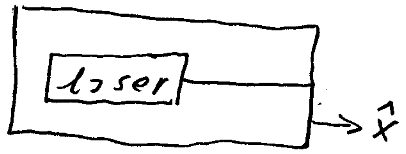
$$= \frac{kE_0^2}{\omega\mu_0}\sin^2(kx-\omega t)\hat{x}$$

Tidsmedelvärdet av $\bar{P}$ ges av

$$\langle\bar{P}\rangle = \frac{1}{T}\int_0^T \frac{kE_0^2}{\omega\mu_0}\underbrace{\sin^2(kx-\omega t)}_{\frac{1}{2}[1-\underbrace{\cos 2(kx-\omega t)}_{\Rightarrow 0 \text{ ty 2 perioder}}]}\hat{x}\,dt = \frac{1}{T}\frac{kE_0^2}{\omega\mu_0}\cdot\frac{1}{2}T\hat{x} = \frac{kE_0^2}{2\omega\mu_0}\hat{x}$$

Periodtiden

Lägg en sluten yta runt lasern

S 

övriga delen av S

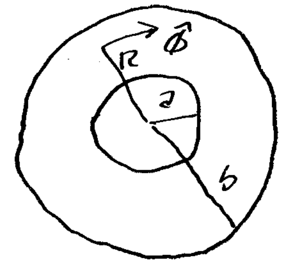
$$P = \oint_S \langle\bar{P}\rangle\cdot d\bar{S} = 0 + \langle\bar{P}\rangle\cdot\hat{x}\,S_0 = \frac{kE_0^2}{2\omega\mu_0}S_0$$

$$\Rightarrow E_0 = \sqrt{\frac{2\omega\mu_0 P}{kS_0}} \approx \sqrt{\frac{2\cdot 3\cdot 10^8\cdot 4\pi\cdot 10^{-7}\cdot 1}{20\cdot 10^{-6}}} \approx 6140\ \text{V/m}$$

Svar: a) $\bar{B} = \frac{kE_0}{\omega}\sin(kx-\omega t)\hat{z}$

b) $E_0 = \sqrt{\frac{2\omega\mu_0 P}{kS_0}} \approx 6.1\ \text{kV/m}$

2, a,



Cirkulationssatsen $\oint_C \bar{H}\cdot d\bar{l} = I_{oms.fri}$

Symmetri $\Rightarrow \bar{H} = H(R)\hat{\phi}$ $\Rightarrow$

$$2\pi R H(R) = NI \Rightarrow H(R) = \frac{NI}{2\pi R}$$

$$W_m = \underset{\text{"Hela } R^3\text{"}}{\int \frac{1}{2}\bar{B}\cdot\bar{H}\,d\tau} = \underset{\text{utanför}}{0} + \int_0^h\int_a^b\int_0^{2\pi} \frac{1}{2}\mu_0\mu_r\left(\frac{NI}{2\pi R}\right)^2 \cdot R\,d\phi\,dR\,dz =$$

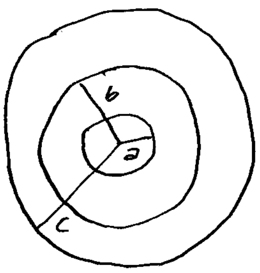
$$= \frac{1}{2}\mu_0\mu_r\left(\frac{NI}{2\pi}\right)^2 \cdot 2\pi \cdot \ln\frac{b}{a}\cdot h = \frac{\mu_0\mu_r N^2 I^2 h}{4\pi}\ln(b/a)$$

b, $W_m \approx 3{,}6\cdot 10^8\,J \approx 100\,kWh$

Svar: a, $W_m = \dfrac{\mu_0\mu_r N^2 I^2 h}{4\pi}\ln(b/a)$

b, $W_m \approx 100\,kWh$

3,



Cylindersymmetri $\Rightarrow \bar{D} = D(R)\hat{R}$

Gauss sats $\oint_S \bar{D}\cdot d\bar{S} = Q_{fri.innes}$

Ansätt laddningen Q på innerledaren och längden L. $\Rightarrow$

$$\oint_S \bar{D}\cdot d\bar{S} = \underset{\text{mantelyta}}{2\pi R L D(R)} + \underset{\text{lock botten}}{0 + 0} = Q_{fri.innes} \underset{a<R<c}{=} Q \Rightarrow$$

$$\bar{D} = \frac{Q}{2\pi R L}\hat{R} \quad ; \quad a < R < c \Rightarrow$$

$$\underset{\text{Luft}}{\bar{E}_l} = \frac{Q}{2\pi\varepsilon_0 R L}\hat{R} \quad ; \quad b < R < c \quad ①$$

$$\underset{\text{plexiglas}}{\bar{E}_p} = \frac{Q}{2\pi\varepsilon_0\varepsilon_r R L}\hat{R} \quad ; \quad a < R < b \quad ②$$

Vi ser att E-fältet är svagare i plexiglas än i luft och vill därför ha så lite plexiglas som möjligt. Men vill undvika genomslag i luft. $\Rightarrow$

$$E_{l.max} = \frac{Q}{2\pi\varepsilon_0 b L} \Rightarrow b = \frac{Q}{2\pi\varepsilon_0 L E_{lmax}} \quad ③$$

3 forts

Vi vill även undvika genomslag i plexiglaset $\Rightarrow$

$$E_{p.max} = \frac{Q}{2\pi\varepsilon_0\varepsilon_r aL} \Rightarrow \frac{Q}{L} = 2\pi\varepsilon_0\varepsilon_r a E_{pmax} \quad \text{ger i } ①,②,③$$

$$\bar{E}_l = \varepsilon_r E_{pmax} \frac{a}{R}\hat{R} \qquad b < R < c$$

$$\bar{E}_p = E_{pmax}\frac{a}{R}\hat{R} \qquad a < R < b$$

$$b = \varepsilon_r a \frac{E_{pmax}}{E_{lmax}}$$

a) $\quad b = \varepsilon_r a \dfrac{E_{pmax}}{E_{lmax}} = 20\text{ mm}$

b) $$U = \int_{Ref}^{Akt} -\bar{E}\cdot d\bar{l} = \int_c^a -\bar{E}\cdot\hat{R}\,dR = \int_c^b -\varepsilon_r E_{pmax}\frac{a}{R}\,dR +$$

$$+\int_b^a -E_{pmax}\frac{a}{R}\,dR = \varepsilon_r E_{pmax}\, a \ln c/b + E_{pmax}\, a \ln b/a =$$

$$= a E_{pmax}\left(\varepsilon_r \ln c/b + \ln b/a\right) \approx 8{,}4\cdot 10^4\text{ V}$$

Svar: a) $\quad b = \varepsilon_r a \dfrac{E_{pmax}}{E_{lmax}} = 20\text{mm}$

b) $\quad U = a E_{pmax}\left(\varepsilon_r \ln c/b + \ln b/a\right) \approx 84\text{ kV}$

4/ Anvind $\bar{E}(\bar{r}) = \int \frac{(\bar{r}-\bar{r}')\,dQ'}{4\pi\varepsilon_0 |\bar{r}-\bar{r}'|^3}$ (*)

$\bar{r} = \bar{0}$

$\bar{r}' = a\hat{r}$

$dQ' = \rho_s\, dS' = \rho_s a^2 \sin\theta\, d\theta\, d\phi$

$\hat{r} = \cos\phi\sin\theta\,\hat{x} + \sin\phi\sin\theta\,\hat{y} + \cos\theta\,\hat{z}$

$$\bar{E} = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_0^{\pi/2} \frac{-\cancel{a}[\cos\phi\sin\theta\,\hat{x} + \sin\phi\sin\theta\,\hat{y} + \cos\theta\,\hat{z}]\rho_s \cancel{a^2}\sin\theta\, d\theta\, d\phi}{4\pi\varepsilon_0 \cancel{a^3}} =$$

$$= \left\{ \int_0^{\pi/2} \underbrace{\sin^2\theta}_{\frac{1}{2}(1-\cos 2\theta)}\, d\theta = \left[\frac{1}{2}\theta - \frac{\sin 2\theta}{4}\right]_0^{\pi/2} = \frac{\pi}{4} \;;\; \int_0^{\pi/2} \sin\theta\cos\theta\, d\theta = \left[\frac{\sin^2\theta}{2}\right]_0^{\frac{\pi}{2}} = \frac{1}{2} \right\} =$$

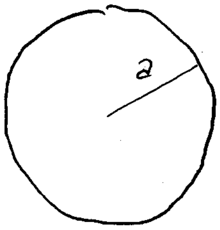
$$= \frac{-\rho_s}{4\pi\varepsilon_0}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\pi}{4}(\cos\phi\,\hat{x} + \sin\phi\,\hat{y}) + \frac{1}{2}\hat{z}\, d\phi =$$

$$= \frac{-\rho_s}{4\pi\varepsilon_0}\left[\frac{\pi}{4}(\sin\phi\,\hat{x} - \cos\phi\,\hat{y}) + \frac{\phi}{2}\hat{z}\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = \frac{-\rho_s}{4\pi\varepsilon_0}\left(\frac{\pi}{4}(2\hat{x} - 0\hat{y}) + \frac{\pi}{2}\hat{z}\right) =$$

$$= \frac{-\rho_s}{8\varepsilon_0}(\hat{x} + \hat{z}) = \left\{ \rho_s = \frac{Q}{\frac{4\pi a^2}{4}} = \frac{Q}{\pi a^2} \right\} = \frac{-Q}{8\pi\varepsilon_0 a^2}(\hat{x} + \hat{z})$$

Svar: $\bar{E}(\bar{0}) = \frac{-Q}{8\pi\varepsilon_0 a^2}(\hat{x} + \hat{z})$

5/



I dethär problemet har vi <u>inte</u> sfärisk symmetri. Vi måste därför ta reda på vilka komponenter fält och strömtätheten har. Givet att $V(r) \Rightarrow$

$$\bar{E} = -\bar{\nabla} V = -\frac{\partial V}{\partial r}\hat{r} \quad \text{dvs} \quad \bar{E} = E(r)\hat{r} \quad \text{men}$$

$$\bar{D} = \varepsilon_0 \varepsilon_r(\theta) E(r)\hat{r} \Rightarrow \bar{D} = D(r,\theta)\hat{r}$$

dvs D-fältet beror av θ. Vi kan inte ansätta en laddning Q jämt fördelad över innerskalet!

Men $\bar{E} = E(r)\hat{r} \Rightarrow \bar{J} = \sigma \bar{E} = \sigma E(r)\hat{r} = J(r)\hat{r}$

Ansätt en ström I från inre till yttre skalet

$$I = \int \bar{J}\cdot d\bar{S} = 4\pi r^2 J(r) \Rightarrow \bar{J} = \frac{I}{4\pi r^2}\hat{r} \Rightarrow$$

$$\bar{E} = \frac{I}{4\pi\sigma r^2}\hat{r}, \quad \bar{D} = \frac{\varepsilon_0\varepsilon_1(1+\cos^2\theta)I}{4\pi\sigma r^2}\hat{r}$$

Gauss yta precis utanför innesfären ger dess laddning

$$Q = \oint_S \bar{D}\cdot d\bar{S} = \int_0^{2\pi}\int_0^{\pi} \frac{\varepsilon_0\varepsilon_1(1+\cos^2\theta)I}{4\pi\sigma \cancel{a^2}}\hat{r}\cdot\hat{r}\,\cancel{a^2}\sin\theta\, d\theta\, d\phi =$$

$$= \frac{\varepsilon_0\varepsilon_1 I}{4\pi\sigma}\cdot 2\pi \underbrace{\left[-\cos\theta - \frac{\cos^3\theta}{3}\right]_0^{\pi}}_{8/3} = \frac{4\varepsilon_0\varepsilon_1 I}{3\sigma}$$

$$U = \int_b^a -\bar{E}\cdot d\bar{l} = \int_b^a -\frac{I}{4\pi\sigma r^2}\hat{r}\cdot\hat{r}\,dr = \frac{I}{4\pi\sigma}\left(\frac{1}{a}-\frac{1}{b}\right) = \frac{I}{4\pi\sigma}\cdot\frac{b-a}{ab}$$

Svar $C = Q/U = \frac{16\pi\varepsilon_0\varepsilon_1}{3}\cdot\frac{ab}{b-a}$