TDDD14/TDDD85
Formal Languages and Automata Theory
2018-09-01

Materials allowed (Tillåtna hjälpmedel):

- A sheet of notes - 2-sided A5 or 1-sided A4. These notes must be handed in together with the answers and signed in the same way as the exam papers. (Ett blad med anteckningar - 2-sidigt A5 eller 1-sidigt A4. Detta blad ska lämnas in med svaren och signeras på samma sätt som övriga papper.)
- An english dictionary. (Engelsk ordbok).

Instructions:

- You may answer in english or swedish.
- Make sure your text and figures are big and clear enough to read easily.
- All answers must be motivated. A correct answer without reasonable motivation may result in zero points!

Grading: The maximum number of points is 34. The grades are as follows:

grade	TDDD14	TDDD85
3:	18–24 p.	15–21 p.
4:	25–29 p.	22–27 p.
5:	30–34 p.	28–34 p.

Problems

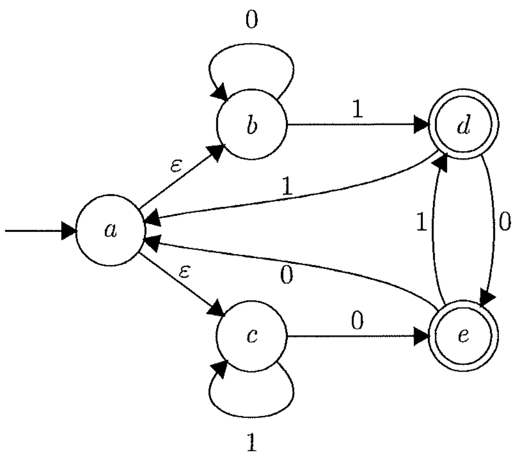
1. Provide regular expressions for the following languages and explain why they are correct. (6 p)

   (a) All strings over $\{0, 1\}$ not beginning with 01.

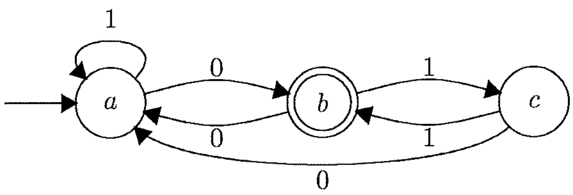
   (b) All strings over $\{a, b, c\}$ with an even number of c's.

   (c) All strings over $\{a, b\}$ not containing ba as a substring.

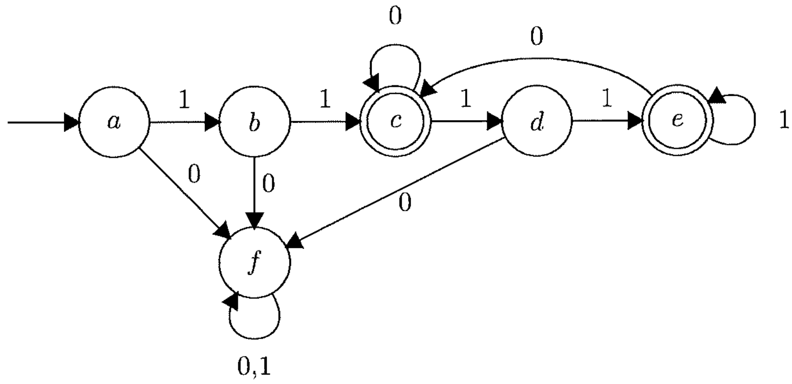
2. Convert the following NFA to an equivalent DFA, using the subset construction method. (4 p)



3. Convert the following DFA to a regular expression using the GNFA method (or one of the other standard methods in the course). (4 p)



4. Use the minimization algorithm from the course to construct a DFA with a minimal number of states that is equivalent to the DFA below. Clearly explain how you get the markings in the table and in which iterations the markings appear. State the resulting equivalence classes and draw the resulting minimal DFA. (4 p)



5. Consider the sequence of all square integers from 2 upwards, i.e. the sequence $2, 4, 16, 256, \ldots$. Note that $2 = 2^1$ and $(2^i)^2 = 2^{2i}$, so the sequence can alternatively be written as $2^1, 2^2, 2^4, 2^8, \ldots$. Now consider the language L that contains all numbers of this sequence in binary, i.e. L contains the sequence $10, 100, 10000, 100000000, \ldots$. Note that 2^i is represented by the string 10^i in binary. (4 p)

Either prove that L is a regular language or use the pumping lemma for regular languages to prove that L is not regular.

6. For each of the following claims, decide if it is true or false, and explain why. (8 p)

   (a) The empty language $\varnothing$ is regular.

   (b) Suppose that L is a regular language and that u, v and w are strings such that L contains the strings uw, uvw and uv^2w. Then L must also contain the string uv^7w.

   (c) The union of a regular language and a context-free language is always context free.

   (d) The intersection of a regular language and a context-free language is always regular.

7. (4 p)

(a) Give an example of two languages L_1 and L_2 such that

$$L_1 \cup L_2 \leq_m L_1 \cap L_2,$$

or prove that there are no such languages.

(b) Give an example of two languages L_1 and L_2 such that

$$L_1 \cap L_2 \leq_m L_1 \cup L_2,$$

or prove that there are no such languages.