TDDD14/TDDD85

Formal Languages and Automata Theory

2017-10-27

Materials allowed (Tillåtna hjälpmedel):

- A sheet of notes - 2-sided A5 or 1-sided A4. These notes must be handed in together with the answers and signed in the same way as the exam papers. (Ett blad med anteckningar - 2-sidigt A5 eller 1-sidigt A4. Detta blad ska lämnas in med svaren och signeras på samma sätt som övriga papper.)
- An english dictionary. (Engelsk ordbok).

Instructions:

- You may answer in english or swedish.
- Make sure your text and figures are big and clear enough to read easily.
- All answers must be motivated. A correct answer without reasonable motivation may result in zero points!

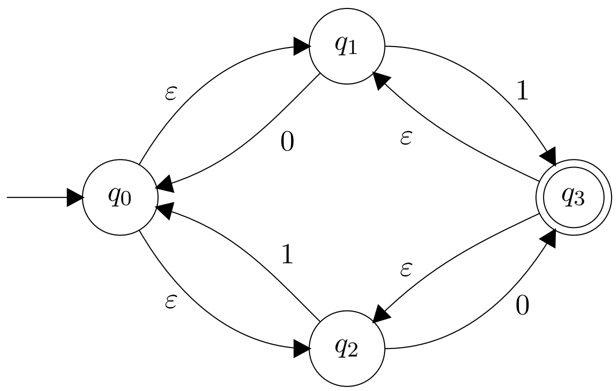
Grading: The maximum number of points is 34. The grades are as follows:

grade	TDDD14	TDDD85
3:	18–24 p.	15–21 p.
4:	25–29 p.	22–27 p.
5:	30–34 p.	28–34 p.

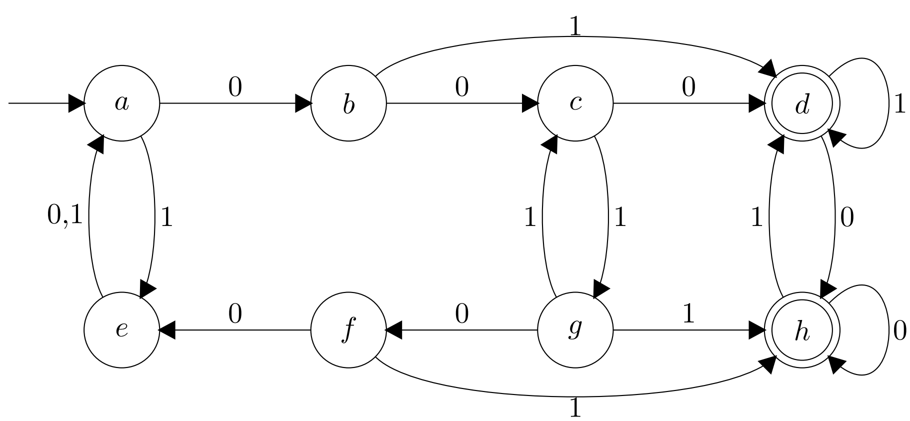
Problems

1. Assume the alphabet $\Sigma = \{0, 1, 2\}$. A string $x_1x_2 \dots x_n$ over Σ^* is in numerical order if $x_i \leq x_{i+1}$ for all i where $1 \leq i < n$. For example, the strings 0001222 and 11122 are in numerical order, but the strings 001102 and 1012 are not in numerical order. Draw the state transition diagram for a DFA that accepts exactly those non-empty strings over Σ^* that are *not* in numerical order. (4 p)

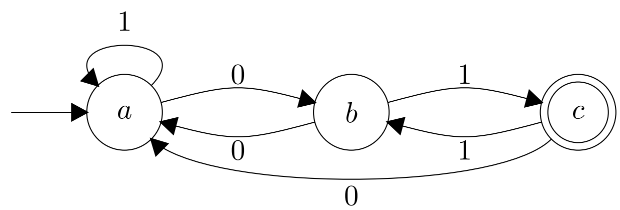
2. Construct a DFA that is equivalent to the following NFA using the subset construction method. You must give both the transition table and the state diagram for the resulting DFA. (4 p)



3. Show that the following DFA has a minimal number of states or construct an equivalent DFA with a minimal number of states. In the latter case, also draw the state diagram for the resulting minimal DFA. Use the algorithm from the lectures and specify clearly what is marked in each stage of the algorithm. (4 p)



4. Convert the following DFA to a regular expression using the GNFA method (or one of the other standard methods in the course). (4 p)



5. Consider the alphabet $\Sigma = \{0, 1, \dots, 9\}$ and the language L defined as $L = \{wuv \mid w + u = v\}$, where the substrings w, u and v are interpreted as ordinary integers. For instance, the string $12719 \in L$ since $12 + 7 = 19$, and the string $10^n20^n30^n \in L$ for all $n \geq 0$ (since $1 + 2 = 3$, $10 + 20 = 30$, $100 + 200 = 300$ etc.). (6 p)

   (a) Show that L is not regular by using the pumping lemma for regular languages.

   (b) Show that L is not context free by using the pumping lemma for context free languages.

6. Consider the following two context-free grammars G_1 (with start variable S_1) and G_2 (with start variable S_2): (6 p)

$$
\begin{aligned}
S_1 &\to A \mid B \\
A &\to 0 \mid 1B \\
B &\to A1
\end{aligned}
$$

$$
\begin{aligned}
S_2 &\to C \mid C1 \\
C &\to 0 \mid 1C1
\end{aligned}
$$

   (a) What is the language $L(G_1) \cup L(G_2)$?

   (b) What is the language $L(G_1) \cap L(G_2)$?

   (c) What is the language $L(G_1) \setminus L(G_2)$?

7. Recall the formal specification of a Turing machine in Kozen. A TM M is a 9-tuple $(Q, \Sigma, \Gamma, \vdash, \sqcup, \delta, s, t, r)$, where (4 p)

- Q is the set of states,
- Σ is the input alphabet,
- Γ is the tape alphabet, where $\Sigma \subseteq \Gamma$,
- $\vdash \in \Gamma \setminus \Sigma$ is the left endmarker,
- $\sqcup \in \Gamma \setminus \Sigma$ is the blank symbol,
- $\delta : Q \times \Gamma \leftarrow Q \times \Gamma \times \{L, R\}$,
- $s \in Q$ is the start state,
- $t \in Q$ is the accept state and
- $r \in Q$ is the reject state.

Let $\Sigma = \{0, 1\}$ and $\Gamma = \Sigma \cup \{\vdash, \sqcup\}$ and define the TMs

$$M_1 = (\{q_0, q_1\}, \Sigma, \Gamma, \vdash, \sqcup, \delta_1, q_0, q_1, q_0),$$

and

$$M_2 = (\{q_0, q_1\}, \Sigma, \Gamma, \vdash, \sqcup, \delta_1, q_0, q_0, q_1),$$

where δ_1 is defined as

$$\begin{aligned} \delta_1(q_0, 0) &= (q_0, \sqcup, R) & \qquad \delta_1(q_1, 0) &= (q_0, 0, L) \\ \delta_1(q_0, 1) &= (q_1, 1, R) & \delta_1(q_1, 1) &= (q_1, \sqcup, L) \\ \delta_1(q_o, \sqcup) &= (q_1, 0, L) & \delta_1(q_1, \sqcup) &= (q_0, \sqcup, R) \\ \delta_1(q_o, \vdash) &= (q_1, \vdash, R) & \delta_1(q_1, \vdash) &= (q_0, \vdash, R) \end{aligned}$$

(a) Does M_1 halt on all inputs and what is the language $L(M_1)$ that it recognizes?

(b) Does M_2 halt on all inputs and what is the language $L(M_2)$ that it recognizes?

8. Prove or disprove the following claim. Let A and B be languages. If $A \leq_m B$ and B is context-free, then A must be context free. (2 p)