# Försättsblad till skriftlig tentamen vid Linköpings universitet

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| **Datum för tentamen** | 2019-08-29 |
| **Sal (1)** | TERE(3) |
| **Tid** | 8-12 |
| **Utb. kod** | TDDD65 |
| **Modul** | TEN1 |
| **Utb. kodnamn/benämning**<br>**Modulnamn/benämning** | Introduction to the Theory of Computation<br>Skriftlig tentamen |
| **Institution** | IDA |
| **Antal uppgifter som ingår i tentamen** | 6 |
| **Jour/Kursansvarig**<br>Ange vem som besöker salen | Christer Bäckström |
| **Telefon under skrivtiden** | 0705-840889 |
| **Besöker salen ca klockan** | c:a 9:00 |
| **Kursadministratör/kontaktperson**<br>(namn + tfnr + mailaddress) | Annelie Almquist<br>2934<br>annelie.almquist@liu.se |
| **Tillåtna hjälpmedel** | Ordbok från/mellan engelska och annat språk.<br>Inga övriga hjälpmedel. |
| **Övrigt** | |
| **Antal exemplar i påsen** | |

# TDDD65
# Introduction to the Theory of Computation
2019-08-29

**Materials allowed:** A dictionary from English to any other language is allowed. No other books, notes etc. are allowed and no electronic equipment (calculators, computer, mobile phones etc.) are allowed.

**Grading:** The maximum number of points is 30 and 15 points are required to pass the examination. At least 15 p is required for grade 3, at least 20 p is required for grade 4 and at least 25 p is required for grade 5.

**Please observe the following:**

- Use only one side of each paper.
- Each problem must be solved on a separate paper (or several papers, if necessary. Subproblems of a problem (a, b, c etc.) may be solved on the same page.
- Properly justify all your answers. If you give only an answer without justification, you may get zero points even if the answer is correct.
- Make sure your answers are readable.
- Try to leave space for comments on every page.

**Good luck!**

## Problems

1. Assume the alphabet $\Sigma = \{0, 1, 2\}$. Define the sum $sum(s)$ of a string $s \in \Sigma^*$ such that (6 p)

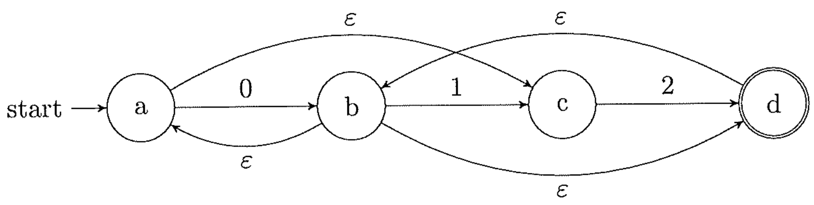
   - $sum(\varepsilon) = 0$ and
   - if $s = x_1, \ldots, x_n$ for some $n > 0$, then $sum(s) = \sum_{i=1}^{n} x_i$,

   i.e. the sum of $s$ is the sum of all its symbols. For example, $sum(0121) = 0 + 1 + 2 + 1 = 4$ and $sum(101120) = 1 + 0 + 1 + 1 + 2 + 0 = 5$.

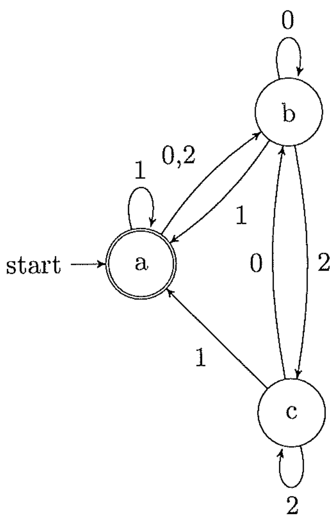
   (a) Draw the state diagram for a DFA that accepts the language $L_1 = \{s \in \Sigma^* \mid sum(s) \text{ is odd } \}$. For instance, 010, 102 and 1121 are in $L_1$, but 0, 112 and 2011 are not in $L_1$.

   (b) Draw the state diagram for a DFA that accepts the language $L_2 = \{s \in \Sigma^* \mid sum(s) = 3k \text{ for some integer } k \geq 0\}$, i.e. all strings $s$ such that $sum(s)$ modulo 3 is 0. For instance, 0, 111 and 10221 are in $L_2$, but 11, 0202 and 11011 are not in $L_2$.

2. Convert the following NFA to a DFA using the subset construction method. (4 p)
Also draw the state diagram for the resulting DFA.




3. Convert the following DFA to a regular expression using the GNFA method (4 p)
(or one of the other standard methods in the course).




4. Define the language $L = \{0^k 1 0^m 2 0^n \mid k, m \geq 0 \text{ and } n \leq k + m\}$ over the (6 p)
alphabet $\Sigma = \{0, 1, 2\}$.

   (a) Prove that $L$ is not regular by using the pumping lemma for regular languages.

   (b) Prove that $L$ is a CFL by providing a CFG for it.

5. Let $L_1$ and $L_2$ be two languages. Define $L = L_1 \setminus L_2$, where "\" denotes (6 p)
set difference, i.e. $A \setminus B = A \cap \overline{B}$.

   (a) Prove that $L$ is Turing recognizable if $L_1$ is Turing recognizable and $L_2$ is decidable.

   (b) Prove or explain why $L$ may not be Turing recognizable if $L_1$ is decidable and $L_2$ Turing recognizable.

6. (8 p)

(a) Recall that for every $k \geq 1$, the problem $k$-SAT is the SATISFIABILITY problem restricted to CNF formulae where each literal contains at most $k$ literals. Prove that 5-SAT is NP-complete. You may use the knowledge that 3-SAT is NP-complete. (*Hint:* There is a very simple solution).

(b) Could the method you used in (a) work also for proving that 2-SAT is NP-complete? Explain why or why not.