TDDD65
Introduction to the Theory of Computation
2018-01-02

Materials allowed: A dictionary from English to any other language is allowed. No other books, notes etc. are allowed and no electronic equipment (calculators, computer, mobile phones etc.) are allowed.

Questions: Christer Bäckström is available on phone 0705-840889 during the exam.

Grading: The maximum number of points is 30 and 15 points are required to pass the examination. At least 15 p is required for grade 3, at least 20 p is required for grade 4 and at least 25 p is required for grade 5.

Results: When the exams are graded there will be an opportunity to see the exams and discuss the result with the examiner (this is called a *tentavisning* in swedish). When and where this will happen will be announced on the course homepage as soon as the grading is finished.

Please observe the following:

- Use only one side of each paper.
- Each problem must be solved on a separate paper (or several papers, if necessary. Subproblems of a problem (a, b, c etc.) may be solved on the same page.
- Properly justify all your answers. If you give only an answer without justification, you may get zero points even if the answer is correct.
- Make sure your answers are readable.
- Try to leave space for comments on every page.

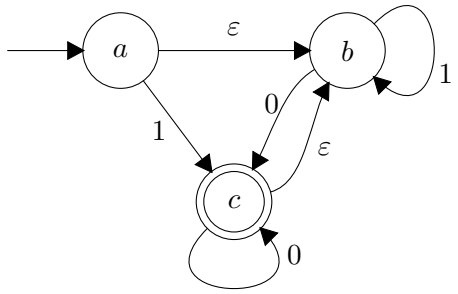
Good luck!

Problems

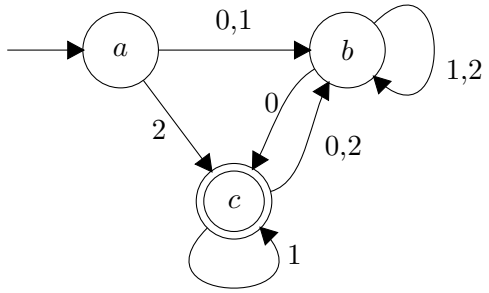
1. Assume the alphabet $\Sigma = \{0, 1\}$. Draw the state diagram for a DFA that accepts the following language (4 p)

$$L = \{0^m 1^n \mid 0 < m < 3 \text{ and } m < n\}.$$

2. Construct a DFA that is equivalent to the following NFA using the subset construction method. You must give both the transition table and the state diagram for the resulting DFA. (4 p)



3. Convert the following DFA to a regular expression using the GNFA method. (4 p)



4. Consider the following context-free grammar G: (6 p)

$$S \to A \mid \varepsilon$$
$$A \to B \mid 1C$$
$$B \to 00A \mid \varepsilon$$
$$C \to 0A \mid 1A$$

   (a) Which of the strings 00000 and 010011 are in $L(G)$?

(b) Is G ambiguous?

(c) Is $L(G)$ regular?

5. Let A, B and C be languages of which we know that $A \leq_m B$ and $A \leq_m C$. (6 p)

   (a) Prove or disprove that $A \leq_m B \cup C$ always holds.

   (b) Prove or disprove that $A \leq_m B \cap C$ always holds.

   (c) Suppose it also holds that $B \leq_m A$. Is it possible that $C \leq_m A$?

6. Let X be some language that can be decided in time $O(n^4)$ and let Y be some language that can be decided in time $O(2^n)$. (6 p)

   (a) Is X in NP?

   (b) Is X in coNP?

   (c) Is Y in NP?

   (d) Is Y NP-hard?

   It is essential that you explain your answers. Just an answer gives zero points!