TDDD65
Introduction to the Theory of Computation
2016-10-26

Materials allowed: A dictionary from English to any other language is allowed. No other books, notes etc. are allowed and no electronic equipment (calculators, computer, mobile phones etc.) are allowed.

Questions: Christer Bäckström is available on phone 0705-840889 during the exam.

Grading: The maximum number of points is 30 and 15 points are required to pass the examination. At least 15 p is required for grade 3, at least 20 p is required for grade 4 and at least 25 p is required for grade 5.

Results: When the exams are graded there will be an opportunity to see the exams and discuss the result with the examiner (this is called a *tentavisning* in swedish). When and where this will happen will be announced on the course homepage as soon as the grading is finished.

Please observe the following:

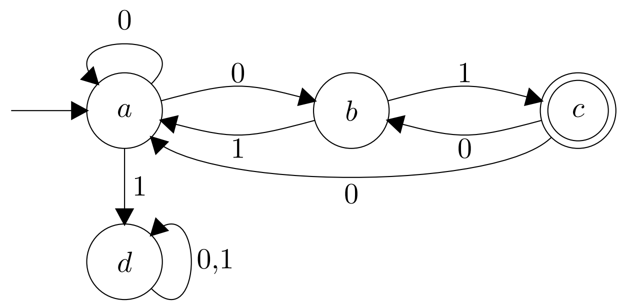
- Use only one side of each paper.
- Each problem must be solved on a separate paper (or several papers, if necessary. Subproblems of a problem (a, b, c etc.) may be solved on the same page.
- Properly justify all your answers. If you give only an answer without justification, you may get zero points even if the answer is correct.
- Make sure your answers are readable.
- Try to leave space for comments on every page.

Good luck!

Problems

1. Assume the alphabet $\Sigma = \{0, 1\}$. Draw the state transition diagram for a DFA that accepts exactly those strings over Σ^* that do not end with three or more 1's. For example, the DFA should accept 0110010 and 1111011 but not 1000111 or 01111. (4 p)

2. Consider the following NFA. (6 p)



(a) Convert the NFA to an equivalent DFA, using the standard method.

(b) Give a regular expression for the language accepted by this NFA. The expression should be reasonably simple.

3. Let L_1 and L_2 be languages over some alphabet Σ. Define the difference language $L = L_1 \setminus L_2$, i.e. L contains all strings in L_1 that are not also in L_2. Show that L must be regular if L_1 and L_2 are regular. (4 p)

4. Consider a language L defined by the following CFG. (6 p)

$$S \to A \,|\, B \,|\, 00 \,|\, 11$$
$$A \to 0A \,|\, 1A \,|\, 1$$
$$B \to 0B \,|\, 1B \,|\, 0$$

(a) What is the language generated by this grammar?

(b) Show that this grammar is ambiguous.

(c) Give an equivalent grammar that is unambigious.

5. Consider the polynomial-time mapping reduction $\leq_m^p$. Prove or disprove each of the following claims: (6 p)

(a) For all languages A, it holds that $A \leq_m^p A$ (i.e. $\leq_m^p$ is reflexive).

(b) For all languages A and B, if $A \leq_m^p B$ then $B \leq_m^p A$ (i.e. $\leq_m^p$ is symmetric).

(c) For all languages A, B and C, if $A \leq_m^p B$ and $B \leq_m^p C$ then $A \leq_m^p C$ (i.e. $\leq_m^p$ is transitive).

6. Suppose we could prove that SAT requires $O(2^c n)$ time, for some constant c, i.e. that SAT requires exponential time in the worst case. Explain what implications this would have for the time complexity of the other NP-complete problems, or why it would not have any such implications. (4)