TDDD65
Introduction to the Theory of Computation
2016-08-27

Materials allowed: A dictionary from English to any other language is allowed. No other books, notes etc. are allowed and no electronic equipment (calculators, computer, mobile phones etc.) are allowed.

Questions: Christer Bäckström is available on phone 0705-840889 during the exam.

Grading: The maximum number of points is 30 and 15 points are required to pass the examination. At least 15 p is required for grade 3, at least 20 p is required for grade 4 and at least 25 p is required for grade 5.

Results: When the exams are graded there will be an opportunity to see the exams and discuss the result with the examiner (this is called a *tentavisning* in swedish). When and where this will happen will be announced on the course homepage as soon as the grading is finished.

Please observe the following:

- Use only one side of each paper.
- Each problem must be solved on a separate paper (or several papers, if necessary. Subproblems of a problem (a, b, c etc.) may be solved on the same page.
- Properly justify all your answers. If you give only an answer without justification, you may get zero points even if the answer is correct.
- Make sure your answers are readable.
- Try to leave space for comments on every page.

Good luck!

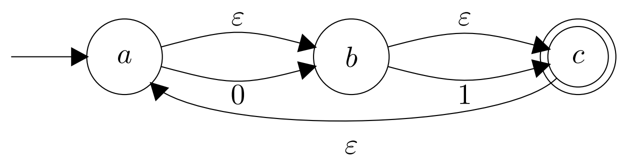
Problems

1. Consider a language L over the alphabet $\Sigma = \{0, 1, 2\}$ where the strings in L are on the form $x_1 x_2 \dots x_n$ where $n \geq 1$ and $x_1 \leq x_2 \leq \cdots \leq x_n$. For example, the strings 0, 001222, 112 and 1111 are in L, but 0121 and 100122 are not in L. (6 p)

   (a) Draw the state diagram for a DFA that recognizes L.

   (b) Define L by a regular expression. (You do *not* need to convert the DFA in your previous answer).

2. Convert the following NFA to an equivalent DFA, using the standard method. (4 p)



3. Give a context-free grammar for each of the following languages L: (6 p)

   (a) L is the same language as in problem 1,

   (b) $L = \{0^n 1^m 2^{n-m} \mid 1 \leq m \leq n\}$.

4. Consider the language (6 p)

$$L = \{0^n 1^{3n} \mid 1 \leq n\}.$$

   Show that L is not regular using the pumping lemma for regular languages.

5. Prove that the function $2^{(\log n)^2}$ grows faster than all polynomials but slower than the exponential functions, by proving the following two claims: (4 p)

   (a) $n^k \in O(2^{(\log n)^2})$ for all $k > 0$.

   (b) $2^{(\log n)^2} \in O(2^{kn})$ for all $k > 0$.

6. Is it possible that P $\neq$ NP but there are NP-complete problems that do not require exponential time to solve? Motivate your answer as well as you can. (4 p)