TDDD65
Introduction to the Theory of Computation
2016-01-07

Materials allowed: A dictionary from English to any other language is allowed. No other books, notes etc. are allowed and no electronic equipment (calculators, computer, mobile phones etc.) are allowed.

Questions: Christer Bäckström will show up after approx one hour and is otherwise available on phone 0705-840889

Grading: The maximum number of points is 30 and 15 points are required to pass the examination. At least 15 p is required for grade 3, at least 20 p is required for grade 4 and at least 25 p is required for grade 5.

Results: When the exams are graded there will be an opportunity to see the exams and discuss the result with the examiner (this is called a *tentavisning* in swedish). When and where this will happen will be announced on the course homepage as soon as the grading is finished.

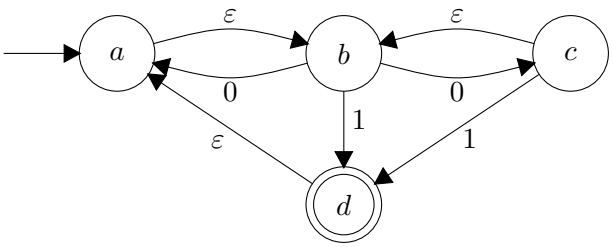
Please observe the following:

- Use only one side of each paper.
- Each problem must be solved on a separate paper (or several papers, if necessary. Subproblems of a problem (a, b, c etc.) may be solved on the same page.
- Properly justify all your answers. If you give only an answer without justification, you may get zero points even if the answer is correct.
- Make sure your answers are readable.
- Try to leave space for comments on every page.

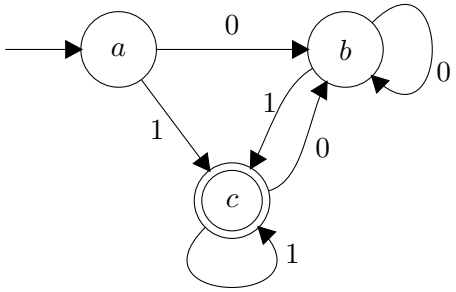
Good luck!

Problems

1. Convert the following NFA to an equivalent DFA, using the standard method. (4 p)



2. Convert the following DFA to an equivalent regular expression using the GNFA method. (4 p)



3. Consider the following two context-free grammars G_1 (with start variable S_1) and G_2 (with start variable S_2): (4 p)

$$S_1 \to A \mid B$$
$$A \to 0 \mid 1B$$
$$B \to A1$$

$$S_2 \to C \mid C1$$
$$C \to 0 \mid 1C1$$

Are these two grammars equivalent, i.e. does $L(G_1) = L(G_2)$ hold or not? Motivate your answer as well as you can.

4. The Thue-Morse sequence is a sequence $w_1, w_2, w_3, \ldots$ of strings over the alphabet $\{0, 1\}$ defined as follows: (6 p)

$$w_1 = 0$$

$$w_{i+1} = w_i \cdot \overline{w_i} \quad \text{for} \quad i > 1$$

where $\overline{w}$ denotes the complement of w, i.e. 0s are replaced by 1s and 1s are replaced by 0s. The first four strings are, thus:

$$w_1 = 0,\ w_2 = 0 \cdot \overline{0} = 01,\ w_3 = 01 \cdot \overline{01} = 0110,\ w_4 = 0110 \cdot \overline{0110} = 01101001.$$

Let L be the language of all strings in the Thue-Morse sequence, that is, $L = \{w_i \mid i \geq 1\}$.

(a) Use the pumping lemma for regular languages to show that L is not regular.

(b) Use the pumping lemma for context-free languages to show that L is not context free.

(Hint: Note that $|w_i| = 2^{i-1}$ for all $i \geq 1$).

5. Prove or disprove the following claims: (6 p)

   (a) $(n + d)^k$ is $O(n^k)$ for all constants $d \geq 1$ and $k \geq 1$.

   (b) $n^{\sqrt{n}}$ is $O((\sqrt{n})^n)$.

   (c) $n^{\log n}$ is $O((\log n)^n)$.

6. Consider the following problem: (6 p)

   HALF-SAT
   Input: A logical formula φ in CNF with an even number of unique clauses (i.e. there are no multiple copies of any clause).
   Question: Is there an assignment that satisfies exactly half of the clauses (no more, no less)?

   Prove that problem HALF-SAT is NP-complete.