TDDD65
Introduction to the Theory of Computation
2015-10-28, kl. 8-12, Room T2

Materials allowed: A dictionary from English to any other language is allowed. No other books, notes etc. are allowed and no electronic equipment (calculators, computer, mobile phones etc.) are allowed.

Questions: Christer Bäckström will show up after approx one hour and is otherwise available on phone 0705-840889

Grading: The maximum number of points is 30 and 15 points are required to pass the examination. At least 15 p is required for grade 3, at least 20 p is required for grade 4 and at least 25 p is required for grade 5.

Results: When the exams are graded there will be an opportunity to see the exams and discuss the result with the examiner (this is called a *tentavisning* in swedish). When and where this will happen will be announced on the course homepage as soon as the grading is finished.

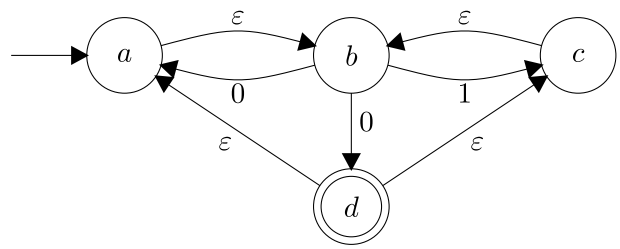
Please observe the following:

- Use only one side of each paper.
- Each problem must be solved on a separate paper (or several papers, if necessary. Subproblems of a problem (a, b, c etc.) may be solved on the same page.
- Properly justify all your answers. If you give only an answer without justification, you may get zero points even if the answer is correct.
- Make sure your answers are readable.
- Try to leave space for comments on every page.

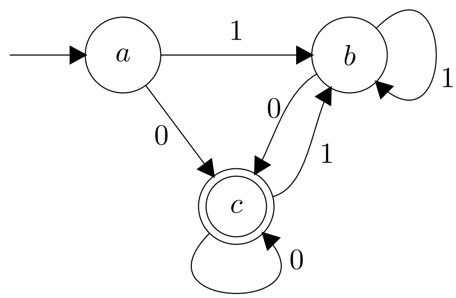
Good luck!

Problems

1. Convert the following NFA to an equivalent DFA, using the standard method. (4 p)



2. Convert the following DFA to an equivalent regular expression using the GNFA method. (4 p)



3. Consider the following context-free grammar G: (6 p)

$$
\begin{aligned}
S &\rightarrow A \mid B \\
A &\rightarrow AB \mid 1C \mid 0 \\
B &\rightarrow 1C \mid 0 \\
C &\rightarrow 1C \mid 0
\end{aligned}
$$

   (a) Show that G is ambiguous.
   (b) Give a non-ambiguous context-free grammar G' such that $L(G) = L(G')$.
   (c) Show that $L(G)$ is regular by giving an equivalent regular expression.

4. Consider the alphabet $\Sigma = \{0, 1, \ldots, 9\}$ and the language L defined as $L = \{wuv \mid w + u = v\}$, where the substrings w, u and v are interpreted as ordinary integers. For instance, the string $12719 \in L$ since $12 + 7 = 19$, and the string $10^n20^n30^n \in L$ for all $n \geq 0$ (since $1 + 2 = 3$, $10 + 20 = 30$, $100 + 200 = 300$ etc.). Show that L is not regular. (4 p)

5. Show the following claims: (6 p)

   (a) $2^{\log(cn)}$ is $O(n)$ for all constants $c > 0$.
   (b) n^k is $O(k^n)$ for all $k \geq 2$.
   (c) $n^{\sqrt{k}}$ is $O((\sqrt{n})^k)$ for $k \geq 4$.

6. Let $G = \langle V, E \rangle$ be a graph, where V is the set of vertices and E is the set of edges. A *cycle* in G is a sequence $v_0, v_1, \ldots, v_n$ of vertices such that (6 p)

   - $v_0 = v_n$ and
   - there is an edge $\{v_{i-1}, v_i\} \in E$ for all i ($1 \leq i \leq n$).

   The cycle is a *Hamilton cycle* if every vertex in V occurs exactly once in the cycle (except that $v_0 = v_n$).

   A weight function $w : E \to \mathbb{Z}_+$ can be used to assign weights to the edges of the graph (where $\mathbb{Z}_+$ denotes the positive integers). Let $v_0, v_1, \ldots, v_n$ be a cycle and let $e_1, \ldots, e_n$ be the edges in the cycle, i.e. $e_1 = \{v_0, v_1\}$, $e_2 = \{v_1, v_2\}$ etc. Then the weight of the cycle is the sum of its edge weights, i.e. $w(e_1) + \cdots + w(e_n)$.

   Consider the following problem, which is NP-complete.

   > HAMILTON CYCLE
   > *Input:* A graph $G = \langle V, E \rangle$.
   > *Question:* Does G have a Hamilton cycle?

   Then consider the following problem:

   > TRAVELLING SALESPERSON (TSP)
   > *Input:* A graph $G = \langle V, E \rangle$, a weight function $w : E \to \mathbb{Z}_+$ and a positive integer k.
   > *Question:* Does G have a Hamilton cycle of weight at most k?

   Show that the TRAVELLING SALESPERSON problem is NP-complete. You may use the knowledge that HAMILTON CYCLE is NP-complete.
   (Hint: Note that the problems are very similar).