4 5

TDDD65/TDDC95 Introduction to the Theory of Computation

2014-10-29, kl. 8–12, Rooms G36/G34

Materials allowed: A dictionary from English to any other language is allowed. No other books, notes etc. are allowed and no electronic equipment (calculators, computer, mobile phones etc.) are allowed.

Questions: Christer Bäckström will show up after approx one hour and is otherwise available on phone 0705-840889

Grading: The maximum number of points is 30 and 15 points are required to pass the examination. At least 15 p is required for grade 3, at least 20 p is required for grade 4 and at least 25 p is required for grade 5.

Results: When the exams are graded there will be an opportunity to see the exams and discuss the result with the examiner (this is called a *tentavisning* in swedish). When and where this will happen will be announced on the course homepage as soon as the grading is finished.

Please observe the following:

- Use only one side of each paper.
- Each problem must be solved on a separate paper (or several papers, if necessary. Subproblems of a problem (a, b, c etc.) may be solved on the same page.
- Properly justify all your answers. If you give only an answer without justification, you may get zero points even if the answer is correct.
- Make sure your answers are readable.
- Try to leave space for comments on every page.

Good luck!

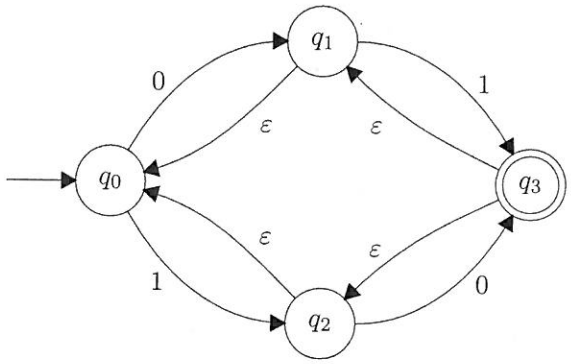
Problems

1. (6 p)

   (a) Let L be the language defined by the regular expression $(01)^*1(10)^*$. Draw the state diagram for a DFA that accepts L.

   (b) Convert this DFA to a regular expression using the GNFA method.

2. Construct a DFA that is equivalent to the following NFA using the standard method. You must give both the transition table and the state diagram for the DFA. (4 p)



3. Consider the following CFG over the alphabet $\{0, 1\}$. (4 p)

$S \to E|T$
$E \to TXE|EXT|TXT|T$
$X \to +|-$
$T \to 0T|1T|\varepsilon$

(a) Show that this grammar is ambiguous.

(b) Give an equivalent CFG that is not ambiguous.

4. Use the pumping lemma to prove that the following language L is not regular. (4 p)

$$L = \{0^n 1^m 2^k \mid 0 \leq m < n < k\}.$$

5. (4 p)

(a) Give an example of two languages L_1 and L_2 such that

$$L_1 \cup L_2 \leq_m L_1 \cap L_2,$$

or prove that there are no such languages.

(b) Give an example of two languages L_1 and L_2 such that

$$L_1 \cap L_2 \leq_m L_1 \cup L_2,$$

or prove that there are no such languages.

6. (8 p)

(a) Recall that for every $k \geq 1$, the problem k-SAT is the SATISFIABILITY problem restricted to CNF formulae where each literal contains at most k literals. Prove that 5-SAT is NP-complete. You may use the knowledge that 3-SAT is NP-complete. (*Hint:* There is a very simple solution).

(b) Could the method you used in (a) work also for proving that 2-SAT is NP-complete? Explain why.