



# Försättsblad till skriftlig tentamen vid Linköpings universitet

(fylls i av ansvarig)

<b>Datum för tentamen</b>	2014-01-09
<b>Sal</b>	TER2
<b>Tid</b>	14-18
<b>Kurskod</b>	TDDD65
<b>Provkod</b>	TEN1
<b>Kursnamn/benämning</b>	Introduction to the Theory of Computing
<b>Institution</b>	<i>IDA</i>
<b>Antal uppgifter som ingår i tentamen</b>	5
<b>Antal sidor på tentamen (inkl. försättsbladet)</b>	3
<b>Jour/Kursansvarig</b>	Christer Bäckström
<b>Telefon under skrivtid</b>	0705-840889
<b>Besöker salen ca kl.</b>	c:a kl. 15
<b>Kursadministratör (namn + tfnr + mailadress)</b>	Liselotte Lundberg, 281278 liselotte.lundberg@liu.se
<b>Tillåtna hjälpmedel</b>	Lexikon från engelska till valfritt annat språk. Inga övriga hjälpmedel.
<b>Övrigt</b> (exempel när resultat kan ses på webben, betygsgränser, visning, övriga salar tentan går i m.m.)	
<b>Vilken typ av papper ska användas, rutigt eller linjerat</b>	
<b>Antal exemplar i påsen</b>	

# TDDD65 Introduction to the Theory of Computation

2014-01-09, kl. 14–18, Room TER2

**Materials allowed:** A dictionary from English to any other language is allowed. No other books, notes etc. are allowed and no electronic equipment (calculators, computer, mobile phones etc.) are allowed.

**Questions:** Christer Bäckström will show up after approx one hour and is otherwise available on phone 0705-840889

**Grading:** The maximum number of points is 30 and 15 points are required to pass the examination. At least 15 p is required for grade 3, at least 20 p is required for grade 4 and at least 25 p is required for grade 5.

**Results:** When the exams are graded there will be an opportunity to see the exams and discuss the result with the examiner (this is called a *tentavisning* in swedish). When and where this will happen will be announced on the course homepage as soon as the grading is finished.

**Please observe the following:**

- Use only one side of each paper.
- Each problem must be solved on a separate paper (or several papers, if necessary. Subproblems of a problem (a, b, c etc.) may be solved on the same page.
- Properly justify all your answers. If you give only an answer without justification, you may get zero points even if the answer is correct.
- Make sure your answers are readable.
- Try to leave space for comments on every page.

**Good luck!**

## Problems

1. Assume the alphabet  $\Sigma = \{0, 1\}$ . (6 p)
  - (a) Draw the state transition diagram for an NFA that accepts exactly those strings over  $\Sigma^*$  that end with at least three 1's. For example, it should accept 0110010111 and 111111 but not 111011.
  - (b) Convert this NFA to a DFA using the standard method.

2. Consider a language  $L$  defined by the following CFG. (6 p)

$$\begin{aligned} S &\leftarrow A|B|0|1 \\ A &\leftarrow 0A|1A|0 \\ B &\leftarrow 0B|1B|1 \end{aligned}$$

- (a) Prove or disprove that this grammar is ambiguous.  
(b) Define  $L$  with a regular expression or prove that  $L$  is not regular.
3. For every positive integer  $k$ , define the language (6 p)

$$L_k = \{0^n 10^n \mid n \leq k\}.$$

Also define the language

$$L_\infty = \bigcup_{k=1}^{\infty} L_k.$$

- (a) Prove that  $L_k$  is regular for every constant value  $k$ .  
(b) Prove that  $L_\infty$  is not regular.
4. Assume some alphabet  $\Sigma$  and two languages  $A, B \subseteq \Sigma^*$ . Define the language (4 p)  
 $A \setminus B = \{x \in \Sigma^* \mid xy \in A \text{ for some } y \in B\}$ . Prove that if both  $A$  and  $B$  are Turing recognizable, then also  $A \setminus B$  is Turing recognizable.
5. For each of the following 3 problems, prove that the problem is in NP (8 p)  
or explain why this cannot be proven (with our current knowledge). For problems in NP, also prove that they are NP-complete, if possible.
- (a) ANDSAT  
Input: Two 3CNF formulae  $\varphi$  and  $\psi$ .  
Question: Are both  $\varphi$  and  $\psi$  satisfiable?
- (b) ORSAT  
Input: Two 3CNF formulae  $\varphi$  and  $\psi$ .  
Question: Is at least one of  $\varphi$  and  $\psi$  satisfiable?
- (c) XORSAT  
Input: Two 3CNF formulae  $\varphi$  and  $\psi$ .  
Question: Is exactly one of  $\varphi$  and  $\psi$  satisfiable?