Försättsblad till skriftlig tentamen vid Linköpings universitet

(fylls i av ansvarig)

Datum för tentamen	2013-10-30
Sal	G35
Tid	8-12
Kurskod	TDDD65
Provkod	TEN1
Kursnamn/benämning	Introduction to the Theory of Computing
Institution	*IDA*
Antal uppgifter som ingår i tentamen	5
Antal sidor på tentamen (inkl. försättsbladet)	3
Jour/Kursansvarig	Christer Bäckström
Telefon under skrivtid	0705-840889
Besöker salen ca kl.	c:a kl. 9
Kursadministratör (namn + tfnnr + mailadress)	Liselotte Lundberg, 281278 liselotte.lundberg@liu.se
Tillåtna hjälpmedel	Lexikon från engelska till valfritt annat språk. Inga övriga hjälpmedel.
Övrigt (exempel när resultat kan ses på webben, betygsgränser, visning, övriga salar tentan går i m.m.)	
Vilken typ av papper ska användas, rutigt eller linjerat	
Antal exemplar i påsen	

TDDD65 Introduction to the Theory of Computation

2013-10-30, kl. 8–12, Room G35

Materials allowed: A dictionary from English to any other language is allowed. No other books, notes etc. are allowed and no electronic equipment (calculators, computer, mobile phones etc.) are allowed.

Questions: Christer Bäckström will show up after approx one hour and is otherwise available on phone 0705-840889

Grading: The maximum number of points is 30 and 15 points are required to pass the examination. At least 15 p is required for grade 3, at least 20 p is required for grade 4 and at least 25 p is required for grade 5.

Results: When the exams are graded there will be an opportunity to see the exams and discuss the result with the examiner (this is called a *tentavisning* in swedish). When and where this will happen will be announced on the course homepage as soon as the grading is finished.

Please observe the following:

- Use only one side of each paper.
- Each problem must be solved on a separate paper (or several papers, if necessary. Subproblems of a problem (a, b, c etc.) may be solved on the same page.
- Properly justify all your answers. If you give only an answer without justification, you may get zero points even if the answer is correct.
- Make sure your answers are readable.
- Try to leave space for comments on every page.

Good luck!

Problems

1. Define the alphabet $\Sigma = \{0, 1\}$ and the language $L \subseteq \Sigma^*$ that consists of all non-empty strings where every symbol occurs at least three times in a row whenever it occurs. For example, the strings 0000111000001111000 and 111100011100000 are in L, but the strings $\underline{00}111000\underline{11}$ and $000\underline{1}00011111$ are not in L (the reasons are underlined). (6 p)

   (a) Give a regular expression defining L. (3)

   (b) Draw the transition graph for a DFA that accepts L. (3)

2. Consider the following CFG G over the alphabet $\Sigma = \{a, b, c, \square, \triangle\}$. (6 p)

$$S \rightarrow S\square S \mid S\triangle S \mid A$$
$$A \rightarrow a \mid b \mid c$$

(a) Prove that G is ambiguous. (3)

(b) Give an equivalent CFG that is not ambiguous. (3)

3. Define the language $L = \{0^m 1 0^n 2 0^{m+n} \mid m, n \geq 0\}$ over the alphabet $\Sigma = \{0, 1, 2\}$. (6 p)

(a) Prove that L is not regular. (3)

(b) Prove that L is a CFL by providing a CFG for it. (3)

4. Consider the mapping reduction $\leq_m$. Prove or disprove each of the following claims: (6 p)

(a) For all languages A, it holds that $A \leq_m A$ (i.e. $\leq_m$ is reflexive). (2)

(b) For all languages A and B, if $A \leq_m B$ then $B \leq_m A$ (i.e. $\leq_m$ is symmetric). (2)

(c) For all languages A, B and C, if $A \leq_m B$ and $B \leq_m C$ then $A \leq_m C$ (i.e. $\leq_m$ is transitive). (2)

5. Prove that the following problem is NP-complete. (6 p)

The **0/1 Integer Program Feasability (0/1-IPF)** problem takes a set of m inequalities over n variables $(x_1, \ldots, x_n)$ on the following form as input

$$\begin{array}{ccccccccc} a_{1,1}x_1 & + & a_{1,2}x_2 & + & \cdots & + & a_{1,n}x_n & \leq & b_1 \\ a_{2,1}x_1 & + & a_{2,2}x_2 & + & \cdots & + & a_{2,n}x_n & \leq & b_2 \\ \vdots & & \vdots & & & & \vdots & & \vdots \\ a_{m,1}x_1 & + & a_{m,2}x_2 & + & \cdots & + & a_{m,n}x_n & \leq & b_m \end{array}$$

where all constants ($a_{i,j}$ and b_i) are integers and the variables can only have value 0 or 1. The problems is to decide if there is an assignment of values to the variables $x_1, \ldots, x_n$ such that all m inequalities hold simultaneously. (Hint: There is a straightforward reduction from **3SAT** to **0/1-IFP** where n is the number of variables and m the number of clauses of the **3SAT** instance.)