TDDD14/TDDD85

Formal Languages and Automata Theory
(Formella språk och automatateori)

2017-05-29

Materials allowed (Tillåtna hjälpmedel):

- A sheet of notes - 2-sided A5 or 1-sided A4. These notes must be handed in together with the answers and signed in the same way as the exam papers.

  (Ett blad med anteckningar - 2-sidigt A5 eller 1-sidigt A4. Detta blad ska lämnas in med svaren och signeras på samma sätt.)

- An english dictionary. (Engelsk ordbok).

Instructions:

- You may answer in english or swedish.
- Make sure your text and figures are big and clear enough to read easily.
- All answers must be motivated. A correct answer without reasonable motivation may result in zero points!

Grading: The maximum number of points is 34. The grades are as follows:

grade	TDDD85	TDDD14
3:	15–21 p.	18–24 p.
4:	22–27 p.	25–29 p.
5:	28–34 p.	30–34 p.

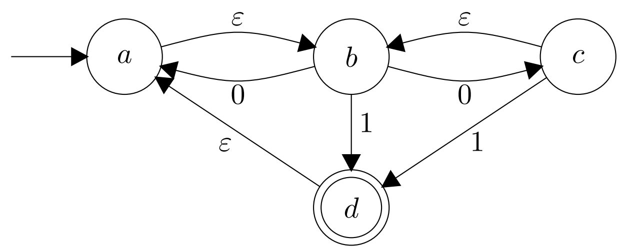
Note that the grading differs between the courses instead of having different problems as previously!

Problems

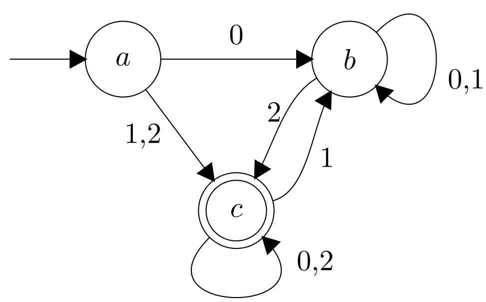
1. Which of the following 4 pairs of regular expressions are equivalent, i.e. denote the same language (4 p)

(a)	$a(bca)^*$	$(abc)a^*$
(b)	$((ab)^*bc)^*ab$	$(ab+bc)^*$
(c)	$((ab)^*ab)^*$	$(ab)^*$
(d)	$(a^*aa)^*$	$(aa)^*$

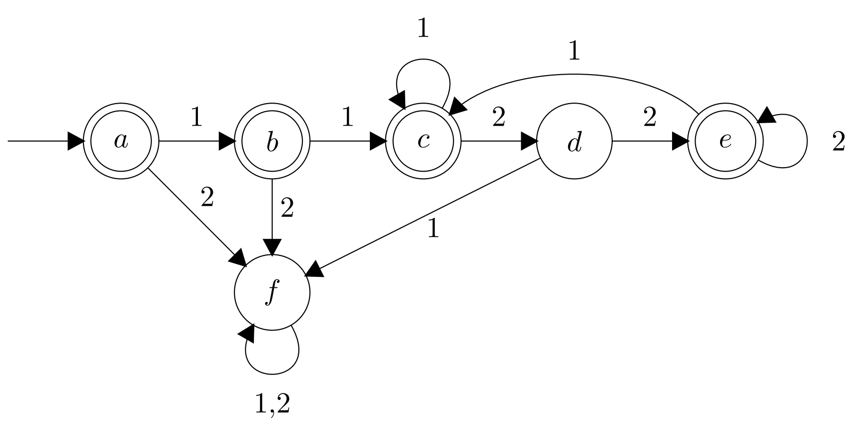
2. Convert the following NFA to an equivalent DFA, using the subset construction method. (4 p)



3. Convert the following DFA to an equivalent regular expression using the GNFA method (or some other standard method from the course). (4 p)



4. Show that the following DFA has a minimal number of states or construct an equivalent DFA with a minimal number of states. (4 p)



5. Give context-free grammars for the following two languages: (4 p)

   (a) $L_{add} = \{a^m b^n c^{m+n} \mid 0 \leq n \leq m\}$
   (b) $L_{sub} = \{a^m b^n c^{m-n} \mid 0 \leq n \leq m\}$

6. (6 p)

   (a) Prove that the following language is not regular, by using the pumping lemma for regular languages.

   $$L_1 = \{(ab)^m (ba)^n \mid 0 < m < n\}$$

   (b) Prove that the following language is not context free by using the pumping lemma for context-free languages.

   $$L_2 = \{w \in \{a, b, c\}^* \mid \#a(w) = \#b(w) \text{ and } \#c(w) = \#a(w) \cdot \#b(w)\}$$

   (Where $\#x(w)$ denotes the number of occurences of symbol x in w).

7. (4 p)

   (a) Let $A = \{a^n b^n \mid n \geq 1\}$, which is not a regular language. Can there be a regular language B such that $A \subseteq B$?
   (b) Let A and B be languages. Suppose A is context free and $A \leq_m B$. Is it possible that B is regular?

8. (4 p)

   (a) Explain why adding a stack to a Turing machine would not make it more powerful, i.e. why it could not recognize more languages, than an ordinary Turing machine.
   (b) In the ordinary Turing-machine model, the tape alphabet must be finite. Would it be possible to define a variant where we allow the tape alphabet to also contain the natural numbers? If so, could such a machine be simulated on an ordinary Turing machine? Clearly state any assumptions you make.