Försättsblad till skriftlig tentamen vid Linköpings universitet

Datum för tentamen	2019-09-07
Sal (1)	TER2(7)
Tid	8-12
Utb. kod	TDDD08
Modul	TEN1
Utb. kodnamn/benämning **Modulnamn/benämning**	Logikprogrammering Skriftlig tentamen
Institution	IDA
Antal uppgifter som ingår i tentamen	10
Jour/Kursansvarig Ange vem som besöker salen	Włodzimierz Drabent wlodzimierz.drabent@liu.se
Telefon under skrivtiden	013 - 28 2017
Besöker salen ca klockan	8:40, 10:45
Kursadministratör/kontaktperson (namn + tfnr + mailaddress)	Veronica Kindeland Gunnarsson veronica.kindeland.gunnarsson@liu.se 013-28 56 34
Tillåtna hjälpmedel	* Ett papper med valfria anteckningar, 1 sida A4 eller 2 sidor A5. Anteckningarna ska signeras på samma sätt som övriga blad och bifogas tentamen vid inlämnandet. * Engelsk ordbok
Övrigt	
Antal exemplar i påsen	

Exam, TDDD08 Logic Programming

2019-09-07, 08:00 – 12:00

Means of assistance (hjälpmedel):
• A sheet of notes – 2 sided A5 or 1 sided A4. The notes should be signed in the same way as the exam sheets and returned together with the exam.
• English dictionary

You may answer in English or Swedish.

Grade limits: 3: 17 p, 4: 23 p, 5: 29 p (for total 35 p).

Remember to give explanations for all answers!
Unexplained answers may be granted 0 points.
For instance, when you write a program you should explain the relations defined by the predicates of the program, and the role of each clause.

GOOD LUCK!

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1. Determine which of the following pairs of terms are unifiable, and provide a most general unifier (mgu) in case there is one.

   a) $p(f(X,Y), f(V,a), g(V), X)$
   $p(Z, Z, W, W)$

   b) $[\,g(X, f(X)), f(a)\,]$
   $[\,g(Y,Z), Z \,|\, Y\,]$

   c) $p(g(X,Y), X, Z)$
   $p(Z, f(V), g(f(a), b))$

   d) $p(X, X, Y, g(Y))$
   $p(f(Z,Z), f(V, g(a)), b, V)$

   (4 points)

2. A programmer compiled a program

```
p :- q( t1 ).
q( t2 ).
```

   where t_1 and t_2 are two non-unifiable terms (without any common variables). In spite of this the Prolog system answered "yes" for the goal ?-p. Explain this. Give an example pair of such terms.

   What should be the result for this program and this query? (In other words: What are the SLD-derivations for this program and this query?) (2 points)

3.(a) Write a logic program defining a predicate $dm/2$ (dm for "double member"), which checks whether a given term occurs at least twice in a given list (as its element).

(b) Write a logic program defining a predicate $ee/2$ (ee for "even equal") which is true when its arguments are lists, say $[t_1, \ldots, t_n]$ and $[s_1, \ldots, s_m]$, and $t_{2i} = s_{2i}$ for any i such that $2i \leq n$ and $2i \leq m$ (in other words: the even numbered elements present in both lists are pairwise equal).

By a list we mean a term of the form $[t_1, \ldots, t_n]$ (equivalently $[t_1|[t_2|\cdots[t_n|[\,]]\cdots]]$). So, for instance, $[1, 2|3]$ is not a list.

Your programs should be definite logic programs. Thus Prolog arithmetic, negation, `( -> ; )`, and other Prolog built-in predicates should not be used. Remember about explanations, see the top of page 1.

(4 points)

4. Consider ordered trees, in which each non-leaf node contains a data item, and has two children. The data item may be an arbitrary term.

(So a leaf does not contain a data item; a smallest tree consists of a single leaf.)

(a) Design a representation of such trees as terms.
Write a program checking that a term represents such tree.

(b) Write a program finding the list of data items on a path of a tree. (We mean a path from the root to a leaf; each such path should be considered.)

(c) Write a program comparing the length of two given lists, whether the first is not shorter that the other one.

(d) Write a program finding the list of data items on the longest path of a tree.
Hint: Use the longest path list for the left subtree of the root, and that for its right subtree.

Remember about explanations, see the top of page 1. Your programs should be definite logic programs. Thus Prolog arithmetic, negation, `( -> ; )`, and other Prolog built-in predicates should not be used.

(4 points)

5. Consider the following definite clause program P:

$$p(\,g(0)\,).\qquad\qquad q(\,f(Z)\,).$$
$$p(X) \leftarrow p(Y), r(Y, X).\qquad\qquad q(\,g(X)\,) \leftarrow q(X).$$
$$r(\,g(Z), f(g(Z))\,).$$

(a) Assume that the vocabulary $\mathcal{A}$ contains one constant 0 and two one-argument function symbols f, g. What is the Herbrand universe $\mathbf{U}_{\mathcal{A}}$ corresponding to $\mathcal{A}$?

(b) Is the Herbrand interpretation

$$I = \{ p(g(0)),\ p(f(g(t))),\ q(f(t)),\ q(g(t)),\ r(t, f(g(t'))) \mid t, t' \in \mathbf{U}_{\mathcal{A}} \}$$

a model of the program?

(c) Find the set $PTR(P)$ of atomic logical consequences of the program. Alternatively, find the least Herbrand model $\mathbf{M}_P$ of the program.

(d) Give an example of a ground atom which is a logical consequence of P, and is not an instance of a unary clause of the program.

(e) Give an example of a ground atom which is not a logical consequence of P; the predicate symbol of the atom should be p.

(f) Give an example of a non-ground atom which is a logical consequence of P, and is not an instance of any unary clause of the program.

(6 points)

6. For a chosen query Q and a chosen subset $P_6 \subseteq P$ of the previous program, construct two SLD-trees – one finite and one infinite. (2 points)

7. Consider the program INSERT (describing removing/adding an element from/to a list):

$$i(X, Ys, [X|Ys]).$$
$$i(X, [Y|Ys], [Y|Zs]) \leftarrow i(X, Ys, Zs).$$

(a) Is INSERT correct with respect to the specification

$$S_0 = \{ i(s, [u_1, \ldots, u_n], [s_1, \ldots, s_{n+1}]) \in \mathbf{B}_{\mathcal{A}} \mid n \geq 0 \} \ ?$$

A brief explanation is sufficient here. ($\mathbf{B}_{\mathcal{A}}$ is the Herbrand base.)

(b) Let $|t|$ stand for the number of (occurrences of) one-argument function symbols in a term t. For instance $|a| = 0$, $|f(a, g(b))| = 1$, $|[\,]| = 0$, $|[g(g(a)), f(b, c)]| = |g(h(a))| = 2$, ...Using a standard method, prove that the program is correct with respect to the specification

$$S = \{ i(s, t, u) \in \mathbf{B}_{\mathcal{A}} \mid |s| + |t| = |u| \}.$$

Note that $|[t|u]| = |t| + |u|$, for any ground terms t, u. (4 points)

8. Translate the following DCG (definite clause grammar) into a Prolog program (using a standard approach).

```
p(a) --> [].
p(s(X)) --> [1], p(X), [1].
p(X) --> [2], p(X).
```

Show that $[1, 2, 1, 1, 1]$ is a member of the language of $p(s^2(a))$ by sketching a proof tree, a successful SLD-derivation, or a derivation of the DCG. (Choose one of three possibilities.)

Describe the language of $p(a)$ defined by the DCG.

(We abbreviate $s(s(a))$ by $s^2(a)$, $s(s(s(a)))$ by $s^3(a)$ and so on.) (3 points)

9. Consider the following general program P_9:

$$p(0).$$
$$p(s(s(X))) \leftarrow \neg p(X).$$

   (a) Draw SLDNF-forests for queries $p(Y)$, $p(s^2(0))$, $p(f^3(0))$. Make it clear, which trees are finitely failed and which leaves are floundered.

   (b) Construct the completion $comp(P_9)$ of the program (except for the equality axioms CET). Explain whether $\neg p(s^2(0))$ is a logical consequence of $comp(P_9)$. Do the SLDNF-forests for $p(Y)$ and for $p(s^2(0))$ provide the corresponding results?

   Hint: It may be useful to remember that from CET it follows that $s(X) = s(Y)$ implies $X = Y$, and that $\forall X\ (0 \neq s(X))$. (5 points)

10. Choose one case from the list below, and explain the notion(s).

   Your explanation should be short but precise, and should show that you understand the notion(s). The chosen notions should not be explained in your sheet of notes.

   (a) Herbrand universe. Herbrand interpretation

   (b) Logical consequence.

   (c) A substitution more general than another one. (Include an example of two substitutions, one more general than the other).

   (d) Computed answer, or computed answer substitution.

   (e) Soundness of SLD-resolution

   (f) Incorrectness diagnosis.

   (g) Constraint solver. Solution of a constraint.

   (h) Closed world assumption. (1 point)