Exam, TDDD08 Logic Programming

2018-08-24, 14:00 – 18:00, TER1

Means of assistance (hjälpmedel):
- A sheet of notes – 2 sided A5 or 1 sided A4. The notes should be signed in the same way as the exam sheets and returned together with the exam.
- English dictionary

You may answer in English or Swedish.

Grade limits: 3: 17 p, 4: 23 p, 5: 29 p (for total 35 p).

Remember to give explanations for all answers!
Unexplained answers may be granted 0 points.
For instance, when you write a program you should explain the relations defined by the predicates of the program, and the role of each clause.

GOOD LUCK!

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1. Determine which of the following pairs of terms are unifiable, and provide a most general unifier (mgu) in case there is one.

   a) $p(X, f(Z), f(V))$
   $p(Y, f(X), Z)$

   b) $p(X, g(Y), X)$
   $p(f(Z), Z, f(g(a))$

   c) $p([X|Z], Z)$
   $p([f(Y), f(Y)], f(a))$

   d) $p(f(Z), X, f(Z))$
   $p(f(X), g(Y, V), V)$

   (4 points)

2. Give a simple example of a program and a query for which the results given by Prolog are different from those described by the theory (SLD-resolution). The difference should be success in practice and failure in theory, or vice versa.

   (2 points)

3. Write a program defining the following predicates:

   (a) $nl/1$ describing the lists of numbers.
   (So $[77, 3]$, $[]$ are such lists, and $[a, 3]$ is not.)

(b) *nlg*/1 describing the lists of numbers in which a number is immediately followed by a greater number.
(So [3, 7], [4, 3, 7, 6] are such lists, and [], [3, 3, 2] are not.)

(c) *nls*/1 describing the lists of distinct numbers, sorted in ascending order.
(So [3, 7, 8], [3], [] are such lists, and [3, 3, 4] is not.)

(d) *top*/2 finding the greatest number in a nonempty list of numbers (which may be not distinct).

Use the Prolog built-in predicate `number`/1, which recognizes that its argument is a number. Use the built-in predicate </2 to compare numbers. Otherwise, your program should be a definite clause logic program; you should not use other Prolog built-in predicates, including negation.

In problem 3d, *top* may fail or produce run-time error when applied to an argument of a wrong kind (not a list of numbers). (4 points)

4. Consider the terms built out of constants a, b and one-argument function symbols f, g.

(a) Define a predicate $t/1$ specifying the set of such terms.

(b) Define a predicate $rep/2$ describing the replacement of each g in such a term by (a one-argument function symbol) h.

(c) Define a predicate $tl/2$ describing the relation that relates any term u (as above) to a list l with as many elements as there are occurrences of f in u.
(The elements of the list may be arbitrary.)

(d) Define a predicate $rP/2$ translating a term (as above) into parentheses-free postfix form, represented as a list. In such form (called also *reverse Polish notation*, r.P.n.) function symbols follow their arguments. For instance $f(g(a))$ in r.P.n. is $[a, g, f]$.

In problem 4d you may use the built-in `append`/3 of Prolog. Otherwise, no Prolog built-ins may be used, your programs should be definite clause logic programs.

To get the full score for problem 4d, your program should avoid unnecessary inefficiencies when translating a ground expression into a list. Hint: begin with a simple, possibly less efficient program.

(5 points)

5. Consider the following definite program P:

$$p(\,f(X), g(X)\,).$$
$$q(a).$$
$$q(f(Z)) \leftarrow q(Y),\ p(\,f(Y), g(Z)\,).$$
$$r(Z) \leftarrow p(\,Z, Z\,).$$

   (a) Assume that the vocabulary $\mathcal{A}$ contains one constant a and two one-argument function symbols f, g. What is the Herbrand universe $\mathbf{U}_\mathcal{A}$ corresponding to $\mathcal{A}$?

   (b) Find the least Herbrand model $\mathbf{M}_P$ of the program. Alternatively, find the set $PTR(P)$ of atomic logical consequences of the program.

   (c) Give an example of a ground atom which is a logical consequence of P; the atom should not be an instance of a unary clause of the program.

   (d) Give an example of a ground atom which is not a logical consequence of P, its predicate symbol should be q.

   (e) Give an example of a non-ground atom which (1) is a logical consequence of P, and (2) is not an instance of any unary clause of the program.

(5 points)

6. For a chosen query Q and a chosen subset $P_2 \subseteq P$ of the previous program construct two SLD-trees (using different selection rules) – one finite and one infinite. (2 points)

7. Consider the program INSERT:

$$i(\,X,\, Ys, [X\,|\,Ys]\,).$$
$$i(\,X, [Y\,|\,Ys], [Y\,|\,Zs]\,) \leftarrow i(X,\, Ys, Zs).$$

   (a) Explain why INSERT is not correct with respect to the specification

$$S_0 = \{\, i(s, [u_1, \ldots, u_n], [s_1, \ldots, s_{n+1}]) \in \mathbf{B}_\mathcal{A} \mid n \geq 0 \,\}.$$

   ($\mathbf{B}_\mathcal{A}$ is the Herbrand base.)

   (b) Let $|t|$ stand for the number of (occurrences of) constants in a term t. For instance $|a| = 1$, $|f(a,b)| = 2$, $|[]| = 1$ (as [] is a constant), $|[a]| = 2$ (as $[a]$ is an abbreviation for $[a|[]]$), $|[a, f(a,a)]| = 4$, etc.

   Using a standard method, prove that the program is correct w.r.t. the specification

$$S = \{\, i(s,t,u) \in \mathbf{B}_\mathcal{A} \mid\ |s| + |t| = |u| \,\}.$$

   Note that $|[t|u]| = |t| + |u|$, for any ground terms t, u. (4 points)

8. Consider the DCG:

```
p(0) --> q.                    q --> [].
p(s(N)) --> [1], q, p(N).      q --> [2], q.
```

(a) Translate the DCG into a Prolog program (using a standard approach).

(b) Show that $[1, 2, 2]$ is a member of the language of $p(s(0))$ by sketching a proof tree or a successful SLD-derivation.

(c) What is the language of $p(s(0))$?

(3 points)

9. Consider the following general program P_4:

$$p(X) \leftarrow q(X).$$
$$p(X) \leftarrow \neg q(X).$$
$$q(a) \leftarrow r(Y).$$
$$r(b).$$

Draw SLDNF-forests for queries $p(b)$ and $p(Y)$. Make it clear which trees are finitely failed, which leaves are floundered, which branches are successful derivations, and what are their answers.

Construct the completion $comp(P_4)$ of the program (except for the equality axioms CET). Explain whether $p(X)$ is a logical consequence of $comp(P_4)$. Does the SLDNF-forest for $p(Y)$ provide a corresponding result?

(5 points)

10. Choose one case from the list below, and explain the notion(s).

Your explanation should be short but precise, and should show that you understand the notions. The chosen notions should not be explained in your sheet of notes.

(a) Interpretation. Herbrand interpretation.

(b) SLD-tree.

(c) Soundness of SLD-resolution.

(d) Incorrectness diagnosis.

(e) Difference list.

(f) Constraint predicate (called also interpreted predicate).

(g) Negation as finite failure.

(h) Nonmonotonic reasoning.

(1 point)