Exam, TDDD08 Logic Programming

2018-01-04, 14:00 – 18:00

Means of assistance (hjälpmedel):
• A sheet of notes – 2 sided A5 or 1 sided A4. The notes should be signed in the same way as the exam sheets and returned together with the exam.
• English dictionary

You may answer in English or Swedish.

Grade limits: 3: 17 p, 4: 23 p, 5: 29 p (for total 35 p).

Remember to give explanations for all answers!
Unexplained answers may be granted 0 points.
For instance, when you write a program you should explain the relations defined by the predicates of the program, and the role of each clause.

GOOD LUCK!

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1. Determine which of the following pairs of terms are unifiable, and provide a most general unifier (mgu) in case there is one.

   a) $p(f(X,Y), f(V,a), g(V), X)$
   $p(Z, Z, W, W)$

   b) $[\,X, f(a)\,]$
   $[\,Y, Z \mid Y\,]$

   c) $p(g(X,Y), X, Z)$
   $p(Z, f(V), g(f(a), b))$

   d) $p(X, X, Y, g(Y))$
   $p(f(Z,Z), f(V, g(a)), b, V)$

   (4 points)

2. Write a program splitting a list l into two lists, so that the second one contains every third element of l and the first one contains all the remaining elements of l. More precisely, the relation to be defined by the program is:

$$\left\{ \begin{array}{l} ([x_1,\ldots,x_{3n}], [x_1,x_2,x_4,x_5,\ldots,x_{3n-2},x_{3n-1}], [x_3,x_6,\ldots,x_{3n}]), \\ ([x_1,\ldots,x_{3n+1}], [x_1,x_2,x_4,x_5,\ldots,x_{3n-2},x_{3n-1},x_{3n+1}], [x_3,x_6,\ldots,x_{3n}]), \\ ([x_1,\ldots,x_{3n+2}], [x_1,x_2,x_4,x_5,\ldots,x_{3n+1},x_{3n+2}], [x_3,x_6,\ldots,x_{3n}]) \end{array} \;\middle|\; n \geq 0 \right\},$$

   where each x_i is a ground term (for $i = 1, \ldots, 3n+2$).

   Your program should be a definite clause program. In particular, built-in predicates of Prolog should not be used. (3 points)

3. Consider binary trees (represented by terms) defined recursively as follows. A tree is of the form `lf`(t) (a leaf), or `tr`(t_l, t_r), where t_l, t_r are trees (the left and right subtrees, respectively). The term t in a leaf `lf`(t) of a tree will be called a *data item* of the tree, it may be an arbitrary term. Write a logic program defining predicates

   - `tree`/1 checking that a term represents a tree (as described above),
   - `rightmost`/2 finding the rightmost data item of a tree,
   - `in`/2 checking that a term is a data item of a tree,
   - `notin`/2 checking that a term is not a data item of a tree,
   - `neighbours`/3 checking that two terms are consecutive data items of a tree (in the natural ordering of the leaves from left to right).

   Use the built-in predicate `dif`/2 for checking inequality of terms in your program for `notin`/2. Otherwise, your program should be a definite clause program; negation, `( -> ; )`, and other Prolog built-in predicates should not be used. (5 points)

4. Explain the difference between the theoretical notion of unification and its implementation in Prolog.

   Give a simple example of a program and a query for which this difference matters. (2 points)

5. Consider the following definite program P:

$$\begin{array}{ll} p(\,f(Z), f(g(Z)\,)\,). & \\ q(\,a\,). & r(X) \leftarrow p(X,Y), q(Y). \\ q(\,f(X)\,). & r(g(X)) \leftarrow p(X,Y), r(Y). \end{array}$$

   (a) Assume that the vocabulary $\mathcal{A}$ contains one constant a and two one-argument function symbols f, g. What is the Herbrand universe $\mathbf{U}_\mathcal{A}$ corresponding to $\mathcal{A}$?

   (b) Is $I = \{\, p(f(t), f(g(t))) \mid t \in \mathbf{U}_\mathcal{A} \,\} \cup \{\, q(t) \mid t \in \mathbf{U}_\mathcal{A} \,\} \cup \{\, r(t) \mid t \in \mathbf{U}_\mathcal{A} \,\}$ a Herbrand model of P?

   (c) Find the least Herbrand model $\mathbf{M}_P$ of the program. Alternatively, find the set $PTR(P)$ of atomic logical consequences of the program.

   (d) Give an example of a ground atom which is a logical consequence of P, its predicate symbol should be r.

   (e) Give an example of a ground atom which is not a logical consequence of P, its predicate symbol should be q or r.

   (f) Give an example of a non-ground atom which is a logical consequence of P, its predicate symbol should be r.

   (6 points)

6. For a chosen query Q and a chosen subset $P' \subseteq P$ of the previous program construct two SLD-trees (using different selection rules) – one finite and one infinite. (2 points)

7. Consider the program INSERT:

$$
\begin{array}{l}
i(\,X,\,Ys,[X|\,Ys]\,).\\
i(\,X,[Y|\,Ys],[Y|Zs]\,) \leftarrow i(X,\,Ys,\,Zs).
\end{array}
$$

   (a) Is INSERT correct with respect to the specification

   $$S_0 = \{\, i(s,[u_1,\ldots,u_n],[s_1,\ldots,s_{n+1}]) \in \mathbf{B}_{\mathcal{A}} \mid n \geq 0 \,\}\ ?$$

   A brief explanation is sufficient here. ($\mathbf{B}_{\mathcal{A}}$ is the Herbrand base.)

   (b) Let $|t|$ stand for the number of (occurrences of) constants in a term t. For instance $|a| = 1$, $|f(a,b)| = 2$, $|[\,]| = 1$ (as $[\,]$ is a constant), $|[a]| = 2$ (as $[a]$ is an abbreviation for $[a|[\,]]$), $|[a, f(a,a)]| = 4$, etc.

   Using a standard method, prove that the program is correct w.r.t. the specification

   $$S = \{\, i(s,t,u) \in \mathbf{B}_{\mathcal{A}} \mid\ |s| + |t| = |u| \,\}.$$

   Note that $|[t|u]| = |t| + |u|$, for any ground terms t, u. (4 points)

8. Consider the DCG:

```
p( [a] ) --> []
p([a,b|L]) --> [c], p([b|L]), [d].
p([b,a|L]) --> [e], p([a|L]), [f].
p([a,a|L]) --> p([a|L]).
```

   (a) Translate the DCG into a Prolog program (using a standard approach).

   (b) Show that $[c, e, f, d]$ is a member of the language of $p([a, a, b, a])$ by sketching a proof tree or a successful SLD-derivation.

   (c) What is the language of $p([a, a, b, a])$ defined by the DCG?

   (3 points)

9. Consider the following general program P_4:

   $$p(s(X)) \leftarrow \neg\, p(X).$$

   (a) Draw SLDNF-forests for queries $p(s^3(0))$ and $p(Y)$. Make it clear which trees are finitely failed, which leaves are floundered, which branches are successful derivations, and what are their answers.

   We abbreviate $s(s(X))$ by $s^2(X)$, $s(s(s(X)))$ by $s^3(X)$ and so on.

   Find out a general pattern – what is the result for $p(s^i(0))$ for $i = 0, 1, \ldots$.

   (A brief explanation is sufficient here.)

(b) Construct the completion $comp(P_4)$ of the program (except for the equality axioms CET). Explain whether $p(s(0))$ is a logical consequence of $comp(P_4)$. Does the SLDNF-tree for $p(Y)$ provide a corresponding result? (5 points)

10. Choose one case from the list below, and explain the notion(s).

Your explanation should be short but precise, and should show that you understand the notions. The chosen notions should not be explained in your sheet of notes.

(a) Model of a set of formulae. Logical consequence.

(b) Resolvent (in SLD-resolution).

(c) Completeness of SLD-resolution.

(d) Incorrectness diagnosis.

(e) Constraint predicate (called also interpreted predicate).

(f) Closed world assumption.

(g) Nonmonotonic reasoning.

(1 point)