A 5

Exam, TDDD08 Logic Programming

2015-01-07, 14:00 – 18:00

Means of assistance (hjälpmedel): A sheet of notes – 2 sided A5 or 1 sided A4. The notes should be signed in the same way as the exam sheets and returned together with the exam.

You may answer in English or Swedish.

Grade limits: 3: 17 p, 4: 23 p, 5: 29 p (for total 35 p).

Remember to give motivations to all answers!
For instance, when you write a program you should explain the relations defined by the predicates of the program, and the role of each clause.

GOOD LUCK!

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1. Determine which of the following pairs of terms that are unifiable, and provide a most general unifier (mgu) in case there is one.

   a) $p(\,X, X, g(V)\,)$
   $p(\,f(Y), f(Z), Y\,)$

   b) $q(\,X,\ Z,\ g(X),\ Z\,)$
   $q(\,f(Y),\ g(Y),\ g(f(a)),\ g(f(a))\,)$

   c) $p(\,f(X), g(X, f(Z))\,)$
   $p(\,Z, g(Y, f(Y))\,)$

   d) $[X, Y|X]$
   $[Z, f(Z), V]$

   (4 points)

2. Explain the difference between the theoretical notion of unification and its implementation in Prolog.

   Does the difference appear in any case of problem 1 above? (2 points)

3. Consider binary trees (represented by terms) defined recursively as follows. A tree is of the form $lf(t)$ (a leaf), where t is an arbitrary term, or $tr(t_l, t_r)$, where t_l, t_r are trees (the left and right subtrees, respectively). Write a logic program defining predicates

   - *tree*/1 checking that a term represents a tree (as described above),
   - *leftmost*/2 finding the leftmost leaf of a tree,
   - *doublel*/1 checking that a tree contains two identical leaves.

   (4 points)

4. Consider nested lists defined recursively as those lists, whose each element is either $e(t)$ or a nested list (where t is an arbitrary term). Write a program flattening such lists – transforming a nested list into a list with all elements of the form $e(t)$, the same as those in the original nested list and in the same order. For instance $\mathit{flatten}(\,[e(a),[e(b),e(c)]\,],\ [e(a),e(b),e(c)]\,)$, and $\mathit{flatten}(\,[\,[e(1)],[e(2),[\,],[e(3),e(4)]\,],e(5)],[\,e(1),e(2),e(3),e(4),e(5)]\,)$ should be answers of the program, and queries $\mathit{flatten}(\,[a,b],Y)$ and $\mathit{flatten}(e(a),Y)$ should fail.

   You may use the standard append/3 built-in. Remember that by a list we mean a term of the form $[t_1,\ldots,t_n]$ (equivalently $[t_1|[t_2|\cdots[t_n|[\,]]\cdots]]$). So for instance $[1,2|3]$ is not a list. (3 points)

5. Consider the following definite program P:

$$
\begin{array}{ll}
p(a,f(b)). & q(a). \\
p(f(X),g(X)). & q(f(a)) \leftarrow q(g(g(X))). \\
 & q(f(b)) \leftarrow q(g(X)). \\
 & q(g(Y)) \leftarrow q(X),\ p(X,f(Y)). \\
 & q(Y) \leftarrow q(g(X)),\ p(f(X),Y).
\end{array}
$$

   Assume that the vocabulary $\mathcal{A}$ contains two constants a,b and two one-argument function symbols f,g. What is the Herbrand universe $\mathbf{U}_{\mathcal{A}}$ corresponding to $\mathcal{A}$?

   Find the least Herbrand model $\mathcal{M}_P$ of the program. Alternatively, find the set $PTR(P)$ of atomic logical consequences of the program.

   Give two examples of atoms which are logical consequences of P and one which is not; the atoms should not be instances of the unary clauses of P. (5 points)

6. Choose a subset $P' \subseteq P$ of the previous program, and for P' and a query $q(X)$ construct two SLD-trees (using different selection rules) – one finite and one infinite. (2 points)

7. Consider the program P:

$$
\begin{array}{l}
p([\,],[\,],[\,]). \\
p([X,Y|T],[X|T_1],[Y|T_2]) \leftarrow p(T,T_1,T_2).
\end{array}
$$

   The task is to prove that (in any answer $p(l,l_1,l_2)$ of the program) all the arguments of p are lists, the first list is of even length, and the elements of the third one are the even elements of the first one.
   By an even element of list $[t_1,\ldots,t_n]$ we mean each t_{2i} (where $1 < 2i \leq n$).

   (a) Express the required property as a specification S.
   (b) Show that the program is correct w.r.t. S. (4 points)

8. Translate the following DCG into a Prolog program (using a standard approach).

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p --> q(a,N),q(b,N).
q(X,s(N)) --> [X], q(X,N).
q(a,0) --> [].
q(b,0) --> [b].
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Show that $[a, b, b]$ is a member of the language of p, by sketching a proof tree, or a successful SLD-derivation.

Explain what is the language of p. Explain for which t the language of $q(a, t)$ is nonempty. (4 points)

9. Consider the following general program P:

$$p(0).$$
$$p(s(s(X))) \leftarrow \neg p(s(X)), \neg p(X).$$

Draw SLDNF-forests for queries $p(s^3(0))$ and $p(Y)$. (Choose a convenient selection rule.) Make it clear, which trees are finitely failed and which leaves are floundered.

Construct the completion $comp(P)$ of the program (except for the equality axioms). Explain whether $\neg p(s(0))$ is a logical consequence of $comp(P)$.

(5 points)

10. Choose two from the notions below and explain them.

Your explanation should be short but precise, and should show that you understand the notions. The chosen notions should not be explained in your sheet of notes.

(a) Query, and a general query.
(b) Interpretation.
(c) Least Herbrand model
(d) Computed answer (for a definite program and a query).
(e) SLD-tree.
(f) Specification
(g) Incompleteness diagnosis.
(h) Constraint predicate (called also interpreted predicate).
(i) Finitely failed SLD-tree.
(j) Negation as finite failure (NAF).

(2 points)

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