Exam in TDDD08 LOGIC PROGRAMMING

Thursday 16 December, 2010, 14:00–18:00, Room U1/U3

No means of assistance (inga hjälpmedel)!
Grading will rely on the following limits (out of max 36):

Grade	3	4	5
Points	≥ 18	≥ 24	≥ 30

Ulf Nilsson can be reached on phone 076–8601935 during the exam.
You may answer in English or in Swedish as you prefer.
REMEMBER TO GIVE MOTIVATIONS TO ALL ANSWERS!!!

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1. Determine if the following pairs of terms are unifiable and give the most general unifier if there is one:

```
| ?- p(f(X,Y),a,g(X)) = p(f(Z,Z),Z,g(b)).
| ?- p(X,f(X),f(f(X))) = p(Y,f(b),f(Z)).
| ?- p(X,f(X,Y)) = p(g(Y),f(Z,Z)).
| ?- [[X|Y]|Y] = [[a]].
```

(4 points)

2. Consider the following Prolog program:

```
rev([],[]).
rev([X|Xs],Zs) :- rev(Xs,Ys), append(Ys,[X],Zs).
append([],Xs,Xs).
append([X|Xs],Ys,[X|Zs]) :- append(Xs,Ys,Zs).
```

Given a goal of the form `:- rev([t1,...,tn],X)`, how many resolution steps are there in the refutation?

(4 points)

3. Assume that we have an alphabet without function symbols containing the constants $\{a, b, c, d\}$ and the predicate symbols $p/1, q/2, r/1$. Let $\Im$ be the Herbrand interpretation:

$$\{p(a), p(b), p(c), q(a, b), q(b, b), q(c, b), q(d, d), q(d, b), r(c), r(d)\}$$

Which of the following formulas are true in $\Im$?

(a) $\exists X \forall Y q(Y, X)$
(b) $\forall X \exists Y (q(X, Y) \rightarrow r(Y))$

(c) $\forall X \exists Y (q(X,Y) \rightarrow p(Y))$

(d) $\exists X (\neg p(X) \vee r(X))$

(4 points)

4. Illustrate by means of an example that general logic programs in general do not have a least Herbrand model.

(4 points)

5. Translate the following DCG into a Prolog program (using the approach used by most Prolog systems):

```
a --> [1], b.
b --> [0], a.
b --> [0].
```

Use the resulting Prolog program to show that the "string" `[1,0,1,0]` is in the language of a (draw the SLD-refutation). (NB: You don't have to draw the full SLD-tree; only the refutation of the tree.)

(4 points)

6. Consider the following Prolog program:

```
p(X) :- q(Y), r(Y, X).
p(X) :- r(X, X).

q(X) :- s(X).

r(a, b).
r(a, c).
r(b, a).
r(d, d).

s(a).
s(b).
s(c).
```

Draw the SLD-tree of the goal `:- p(X)` assuming that Prolog's computation rule is used. What branches of the tree are not explored if the first clause is replaced by the following one?

```
p(X) :- q(Y), !, r(Y, X).
```

(4 points)

7. Consider the following general logic program:

```
p(X) :- q(X), \+ q(f(X)).
q(X) :- r(X).
q(X) :- s(X), \+ r(X).
r(a).
s(a).
```

Draw the SLDNF-forest of the goal `:- p(X)` given that Prolog's computation rule is used. What answers are produced?

(4 points)

8. Mergesort is a sorting method where a list of integers is sorted into ascending order by first dividing the list into two lists of equal (± 1) length which are sorted recursively and where the two sorted lists are finally "merged" together. The following is a not completely correct attempt to implement mergesort:

```
sort([], []).
sort([X|Xs], Ys) :-
      split(Xs, L1, L2),
      sort(L1, S1),
      sort([X|L2], S2),
      merge(S1, S2, Ys).

split([], [], []).
split([X|Xs], [X|Ys], Zs) :-
       split(Xs, Zs, Ys).

merge([], [], []).
merge([], [X|Xs], [X|Xs]).
merge([X|Xs], [], [X|Xs]).
merge([X|Xs], [Y|Ys], [X|Zs]) :-
      X =< Y,
      merge(Xs, [Y|Ys], Zs).
merge([X|Xs], [Y|Ys], [Y|Zs]) :-
      X > Y,
      merge([X|Xs], Ys, Zs).
```

What is wrong with the program? Modify it so that it works.

(4 points)

9. Let P be a definite program. Prove that P has a model iff P has a Herbrand model.

(4 points)