



## Försättsblad till skriftlig tentamen vid Linköpings Universitet

<b>Datum för tentamen</b>	2013-08-26
<b>Sal (1)</b> Om tentan går i flera salar ska du bifoga ett försättsblad till varje sal och <u>ringa in</u> vilken sal som avses	TER2
<b>Tid</b>	8-12
<b>Kurskod</b>	TDDC17
<b>Provkod</b>	TEN1
<b>Kursnamn/benämning</b> <b>Provnamn/benämning</b>	Artificiell intelligens En skriftlig tentamen
<b>Institution</b>	IDA
<b>Antal uppgifter som ingår i tentamen</b>	7
<b>Jour/Kursansvarig</b> Ange vem som besöker salen	Olov Andersson
<b>Telefon under skrivtiden</b>	070-5473343
<b>Besöker salen ca kl.</b>	10:15
<b>Kursadministratör/kontaktperson</b> (namn + tfnr + mailaddress)	Anna Grabska Eklund, ankn. 2362, anna.grabska.eklund@liu.se
<b>Tillåtna hjälpmedel</b>	Miniräknare/Hand calculator
<b>Övrigt</b>	
<b>Vilken typ av papper ska användas, rutigt eller linjerat</b>	Valfritt
<b>Antal exemplar i påsen</b>	

Linköpings Universitet  
Institutionen för Datavetenskap  
Alexander Kleiner

Tentamen  
TDDC17 Artificial Intelligence  
26 August 2013 kl. 8-12

*Points:*

The exam consists of exercises worth 32 points.  
To pass the exam you need 16 points.

*Auxiliary help items:*

Hand calculators.

*Directions:*

You can answer the questions in English or Swedish.  
Use notations and methods that have been discussed in the course.  
In particular, use the definitions, notations and methods in appendices 1-3.  
Make reasonable assumptions when an exercise has been under-specified.  
Begin each exercise on a new page.  
Write only on one side of the paper.  
Write clearly and concisely.

*Jourhavande:* Olov Andersson, 070-5473343. Olov will arrive for questions around 10:15.

**Question 1 (2+2)**

LOGIC

Consider the following logical theory (where  $x, y$  and  $z$  are variables and *nono*, *america*, and *jim* are constants):

$$\forall x [American(x) \wedge Weapon(y) \wedge Sells(x, y, z) \wedge Hostile(z) \Rightarrow Criminal(x)] \quad (1)$$

$$\exists x [Owns(nono, x) \wedge Missile(x)] \quad (2)$$

$$\forall x [Missile(x) \wedge Owns(nono, x) \Rightarrow Sells(jim, x, nono)] \quad (3)$$

$$\forall x [Missile(x) \Rightarrow Weapon(x)] \quad (4)$$

$$\forall x [Enemy(x, america) \Rightarrow Hostile(x)] \quad (5)$$

$$American(jim) \quad (6)$$

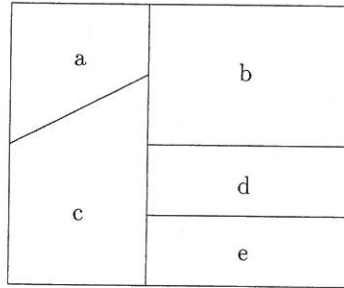
$$Enemy(nono, america) \quad (7)$$

- (a) Convert formulas (1) - (7) into clausal form with the help of appendix 1. [2p]
- (b) Prove that *Criminal(jim)* is a logical consequence of (1) - (7) using the resolution proof procedure. [2p]
- Your answer should be structured using a resolution refutation tree (as used in the book).
  - Since the unifications are trivial, it suffices to simply show the binding lists at each resolution step. Don't forget to substitute as you resolve each step.

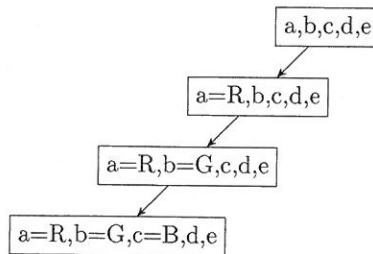
Question 2 (1+2+2)

CONSTRAINT SATISFACTION PROBLEMS

Consider the following map-coloring problem. Five regions  $\{a, b, c, d, e\}$  should be colored by three colors  $\{R, G, B\}$ . No two adjacent regions should have the same color.



- (a) Draw a constraint graph using the regions as the nodes and the constraints as the arcs. [1p]
- (b) Given the partial search tree (below) with the partial assignment  $a = R, b = G, c = B$ , expand the search tree further using backtracking search. Always try the values in the order  $R, G, B$ . Number the nodes according to the sequence in which they would be visited. [2p]



- (c) Please illustrate the process of **forward checking** by filling the available values in the first three steps into the table. [2p]

step	assignment	a	b	c	d	e
0		R,G,B	R,G,B	R,G,B	R,G,B	R,G,B
1	a=R	R				
2	b=G	R	G			
3	c=B	R	G	B		

Question 3 (2+2)

MARKOV DECISION PROBLEMS

$u = 8$	$u = 13$	$u = 12$
$u = 2$	$r = 2$	$u = 10$
$u = 6$	$u = 12$	$u = 11$

Consider the grid world given in the diagram above. The  $u$  values specify the utilities after value iteration has been run to convergence, but the utility of the center state has been removed and instead you only know the reward  $r$  that was associated with that state. Assume that  $\gamma = 0.5$  and that an agent can perform four possible actions: **North**, **South**, **East**, and **West**. With probability 0.2 the agent reaches the intended state. With probability 0.6 it moves to the Left, and with probability 0.2 it moves to the Right of the intended direction at right angles. For example, if it attempts to move East there is a 0.6 probability that it ends up in the square to the north and a 0.2 probability that it ends up in the square to the south. **Hint:** Make use of Appendix 4.

- (a) What is the best action an agent can execute if it is currently in the center state of the grid world? Justify your answer. [2p]
- (b) Compute the utility of the center state. Remember that the other states are already converged. [2p]

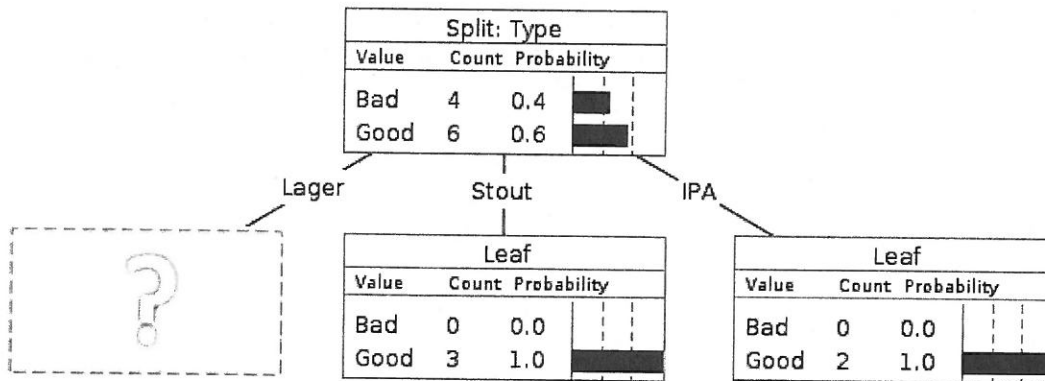
Question 4 (3+1)

DECISION TREES

No	Type	Origin	Mass-market	Taste?
1	Lager	Asia	No	Bad
2	Lager	USA	Yes	Bad
3	Stout	USA	No	Good
4	IPA	USA	No	Good
5	Stout	USA	No	Good
6	Lager	Asia	Yes	Bad
7	Lager	Europe	Yes	Bad
8	Lager	Europe	No	Good
9	IPA	Europe	Yes	Good
10	Stout	Europe	No	Good

Assume that you are helping a restaurant understand a new segment of their customers. Apparently these are very picky about their beer and the restaurant has collected data on various beers to better understand what they want. A small subset of this data is found in the table above.

- (a) Below is an incomplete decision tree which by means of the attributes *Type*, *Origin*, and *Mass-market* attempts to correctly classify whether such a customer will like a beer or not (*Taste?*). Value, Count and Probability in the boxes correspond to the values of the "Taste" target concept (Good/Bad), the number of examples at that point in the tree for each such value, and their proportions. Complete the missing branch of the decision tree *down to its leaves* using the Information Gain strategy (See Appendix) to decide which attribute to split on at each step. Show your calculations! [3p]
- (b) What is *overfitting*? [1p]

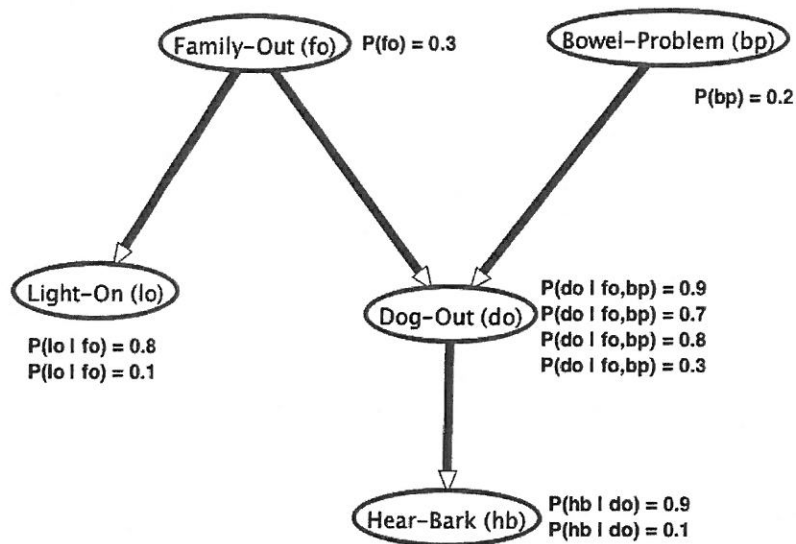


Question 5 (0.5+0.5+3)

BAYESIAN NETWORKS

Use the Bayesian network in the following figure together with the conditional probability tables below for answering the following questions. Appendix 2 may be helpful to use. Here,

- **lo** denotes the event that the outdoor light of the house is on.
- **fo** denotes the event that the family is not at home.
- **bp** denotes the event that the dog of the family has bowel problems.
- **do** denotes the event that the dog is in front of the door.
- **hb** denotes the event that one can hear barking in the vicinity of the house.



fo	P(fo)
T	0.3
F	0.7

bp	P(bp)
T	0.2
F	0.8

fo	lo	P(lo   fo)
T	T	0.8
T	F	0.2
F	T	0.1
F	F	0.9

fo	bp	do	P(do   fo, bp)
T	T	T	0.9
T	T	F	0.1
T	F	T	0.7
T	F	F	0.3
F	T	T	0.8
F	T	F	0.2
F	F	T	0.3
F	F	F	0.7

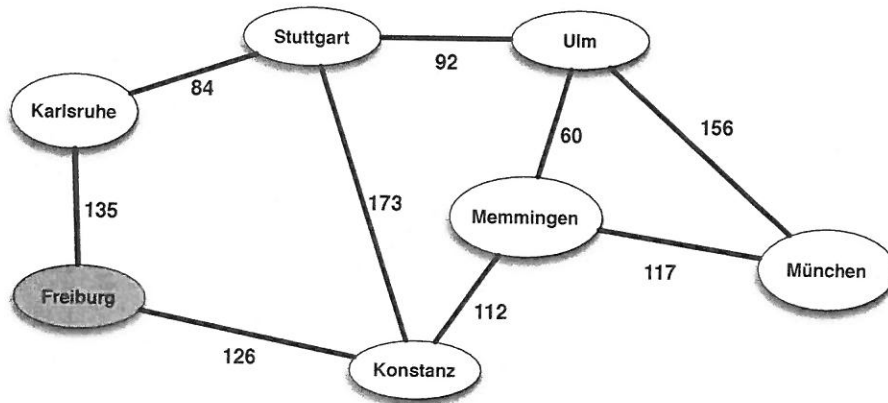
do	hb	P(hb   do)
T	T	0.9
T	F	0.1
F	T	0.1
F	F	0.9

- Write the formula for the full joint probability distribution  $P(lo, fo, bp, do, hb)$  in terms of (conditional) probabilities derived from the shown bayesian network. [0.5p]
- Compute  $P(lo, \neg fo, bp, \neg do, hb)$  [0.5p]
- Compute the probability that the family is out given the light is on and one can hear the dog barking  $P(fo | lo, hb)$ . [3p]

Question 6 (2+1+3)

INFORMED SEARCH

Consider the following road map:



The straight-line distances between **Freiburg** and the other cities are given in the following table:

city	distance
Konstanz	105
Memmingen	174
Stuttgart	216
Karlsruhe	119
Muenchen	278
Ulm	165

- (a) Draw the search tree generated by the  $A^*$  algorithm when searching for a shortest path from München to Freiburg, using as the heuristic the straight-line distance to Freiburg. Include all nodes in your drawing for which  $A^*$  would calculate  $f, g$  and  $h$ . Indicate in which order the nodes are expanded (expanding a node = applying the goal-test and adding its children to the priority queue) and annotate each node with its  $f, g$ , and  $h$  value. [2p]
- (b) Explain what an admissible heuristic function is using the notation and descriptions in (c). [1p]
- (c) Let  $h(n)$  be the estimated cost of the cheapest path from a node  $n$  to the goal. Let  $g(n)$  be the path cost from the start node to  $n$ . Let  $f(n) = g(n) + h(n)$  be the estimated cost of the cheapest solution through  $n$ . Provide a general proof that  $A^*$  using tree-search is optimal if  $h(n)$  is admissible. If possible, use a diagram to structure the proof. [3p]



Question 7 (3+2)

PLANNING

- (a) A partial order causal link (POCL) planner begins with an initial plan and proceeds by resolving *flaws* in this plan. One typically distinguishes between two specific types of flaw. I) Which flaw types exist? II) How is each flaw type *recognized* in a plan? That is, what characterizes a flaw of each type? III) How can each flaw type be resolved? [3p]
- (b) A *planning graph* consists of *levels* of two distinct and alternating types. Explain which information is represented in each level of such a graph. A high-level explanation of each level is sufficient – the exact internal structure of each level does not have to be described. [2p]

## Appendix 1

Converting arbitrary wffs to clause form:

- (a) Eliminate implication signs.
- (b) Reduce scopes of negation signs.
- (c) Standardize variables within the scopes of quantifiers (Each quantifier should have its own unique variable).
- (d) Eliminate existential quantifiers. This may involve introduction of Skolem constants or functions.
- (e) Convert to prenex form by moving all remaining quantifiers to the front of the formula.
- (f) Put the matrix into conjunctive normal form. Two useful rules are:
  - $\omega_1 \vee (\omega_2 \wedge \omega_3) \equiv (\omega_1 \vee \omega_2) \wedge (\omega_1 \vee \omega_3)$
  - $\omega_1 \wedge (\omega_2 \vee \omega_3) \equiv (\omega_1 \wedge \omega_2) \vee (\omega_1 \wedge \omega_3)$
- (g) Eliminate universal quantifiers.
- (h) Eliminate  $\wedge$  symbols.
- (i) Rename variables so that no variable symbol appears in more than one clause.

### Skolemization

Two specific examples. One can of course generalize the technique.

$\exists xP(x)$  :

Skolemized:  $P(c)$  where  $c$  is a fresh constant name.

$\forall x_1, \dots, x_k, \exists yP(y)$  :

Skolemized:  $P(f(x_1, \dots, x_k))$ , where  $f$  is a fresh function name.

## Appendix 2

A generic entry in a joint probability distribution is the probability of a conjunction of particular assignments to each variable, such as  $P(X_1 = x_1 \wedge \dots \wedge X_n = x_n)$ . The notation  $P(x_1, \dots, x_n)$  can be used as an abbreviation for this.

The chain rule states that any entry in the full joint distribution can be represented as a product of conditional probabilities:

$$P(x_1, \dots, x_n) = \prod_{i=1}^n P(x_i \mid x_{i-1}, \dots, x_1) \quad (8)$$

Given the independence assumptions implicit in a Bayesian network a more efficient representation of entries in the full joint distribution may be defined as

$$P(x_1, \dots, x_n) = \prod_{i=1}^n P(x_i \mid \text{parents}(X_i)), \quad (9)$$

where  $\text{parents}(X_i)$  denotes the specific values of the variables in  $\text{Parents}(X_i)$ .

Recall the following definition of a conditional probability:

$$\mathbf{P}(X \mid Y) = \frac{\mathbf{P}(X \wedge Y)}{\mathbf{P}(Y)} \quad (10)$$

The following is a useful general inference procedure:

Let  $X$  be the query variable, let  $\mathbf{E}$  be the set of evidence variables, let  $\mathbf{e}$  be the observed values for them, let  $\mathbf{Y}$  be the remaining unobserved variables and let  $\alpha$  be the normalization constant:

$$\mathbf{P}(X \mid \mathbf{e}) = \alpha \mathbf{P}(X, \mathbf{e}) = \alpha \sum_{\mathbf{y}} \mathbf{P}(X, \mathbf{e}, \mathbf{y}) \quad (11)$$

where the summation is over all possible  $\mathbf{y}$ 's (i.e. all possible combinations of values of the unobserved variables  $\mathbf{Y}$ ).

Equivalently, without the normalization constant:

$$\mathbf{P}(X \mid \mathbf{e}) = \frac{\mathbf{P}(X, \mathbf{e})}{\mathbf{P}(\mathbf{e})} = \frac{\sum_{\mathbf{y}} \mathbf{P}(X, \mathbf{e}, \mathbf{y})}{\sum_{\mathbf{x}} \sum_{\mathbf{y}} \mathbf{P}(\mathbf{x}, \mathbf{e}, \mathbf{y})} \quad (12)$$

## Appendix 3: Decision Tree Learning

### Definition 1

Given a collection  $S$ , containing positive and negative examples of some target concept, the entropy of  $S$  relative to this boolean classification is

$$\text{Entropy}(S) \equiv -p_{\oplus} \log_2 p_{\oplus} - p_{\ominus} \log_2 p_{\ominus},$$

where  $p_{\oplus}$  is the proportion of positive examples in  $S$  and  $p_{\ominus}$  is the proportion of negative examples in  $S$ .

### Definition 2

Given a collection  $S$ , containing positive and negative examples of some target concept, and an attribute  $A$ , the information gain,  $\text{Gain}(S, A)$ , of  $A$  relative to  $S$  is defined as

$$\text{Gain}(S, A) \equiv \text{Entropy}(S) - \sum_{v \in \text{Values}(A)} \frac{|S_v|}{|S|} \text{Entropy}(S_v),$$

where  $\text{values}(A)$  is the set of all possible values for attribute  $A$  and  $S_v$  is the subset of  $S$  for which the attribute  $A$  has value  $v$  (i.e.,  $S_v = \{s \in S \mid A(s) = v\}$ ).

For help in converting from one logarithm base to another (if needed):

$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$\ln x = 2.303 \log_{10} x$$

Note also that for the example, we define  $0 \log 0$  to be 0.

## Appendix 4: Value Iteration

Value iteration is a way to compute the utility  $U(s)$  of all states in a *known* environment (MDP) for the optimal policy  $\pi^*(s)$ .

It is defined as:

$$U(s) = R(s) + \gamma \max_{a \in \mathcal{A}} \sum_{s'} P(s'|s, a) U(s') \quad (13)$$

where  $R(s)$  is the reward function,  $s$  and  $s'$  are states,  $\gamma$  is the discount factor.  $P(s'|s, a)$  is the state transition function, the probability of ending up in state  $s'$  when taking action  $a$  in state  $s$ .

Value iteration is usually done by initializing  $U(s)$  to zero and then sweeping over all states updating  $U(s)$  in several iterations until it converges to the utility of the optimal policy  $U_{\pi^*}(s)$ .

A policy function defines the behavior of the agent, the action  $a$  it will take in each state  $s$ . Once we have computed the utility function of the optimal policy as above,  $U_{\pi^*}(s)$ , we can easily extract the optimal policy  $\pi^*(s)$  itself by simply taking the action that leads to the highest expected utility in each state

$$\pi^*(s) = \arg \max_{a \in \mathcal{A}} \sum_{s'} P(s'|s, a) U_{\pi^*}(s') \quad (14)$$

where „arg max” means selecting the action that maximizes the expression.