LINKÖPINGS UNIVERSITET
Matematiska institutionen
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TATA 57/TATA80 Transform Theory, 2015.08.20, 08-13 TEN 1.

Each question can give 0, 1, 2 or 3 points. An answer is deemed to be good if it is marked with at least 2 points. For grade $n,\ n = 3, 4, 5$, you need $3n - 1$ points and n good answers.
ERASMUS students will have their grades marked according to the scale: A=grade 5, B=grade 4, C=grade 3.
You are allowed to use your own copies of *Transformteori. Sammanfattning, Formler & Lexikon.* No calculators are allowed.
Solutions to the examination can be found on the course homepage after the examination.

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1) Solve the difference equation

$$y(k+2) + 2y(k+1) - 3y(k) = 4 \cdot 3^k$$

with $y(0) = 1,\ y(1) = 1$.

2) Solve the equation

$$y''(t) + y'(t) + 2y(t) = 6te^{-t}$$

for $t \geq 0$ and with $y(0) = y'(0) = 0$.

3) The function $f(t)$ has period 2π and is defined as $f(t) = \begin{cases} 0 & -\pi \leq t < 0 \\ \pi - t & 0 \leq t \leq \pi \end{cases}$.

Determine the Fourier series of $f(t)$s. To which values does the series converge at $t = 0, \pm\pi$? Using the Fourier series, calculate the value of the series

$$\sum_{k=0}^{\infty} \frac{1}{(2k+1)^2}.$$

Give reasons for your working.

4) Find all solutions $y(t) \in L^1(\mathbb{R})$ of the equation

$$\int_0^{\infty} y'(t-u)e^{-u}du = -te^{-2|t|}.$$

5) Determine all functions $y(t)$ with period 2π which satisfy the differential equation

$$y'(t) + 3y(t-\pi) = 2\sin 4t$$

Give reasons for your working.

6) Calculate the value of the integral

$$\int_{-\infty}^{\infty} \frac{\sin\omega}{\omega(\omega^2+4)}\, d\omega.$$

7) Show that the series $f(x) = \sum_{k=1}^{\infty} \frac{1}{(k+x)^2}$ defines a continuous function for all $x \in [0,1]$. Calculate the value of $\int_0^1 f(x)dx$. Give reasons for your working.