Information page for written examinations at Linköping University

Examination date	2019-02-20
Room (1)	TER3(4)
Time	8-12
Edu. code	732A93
Module	TENT
Edu. code name **Module name**	Statistical Methods (Statistical Methods) Examination (Tentamen)
Department	IDA
Number of questions in the examination	5
Teacher responsible/contact person during the exam time	Hector Rodriguez-Deniz
Contact number during the exam time	0728634376
Visit to the examination room approximately	10:00 AM
Name and contact details to the course administrator (name + phone nr + mail)	Ann Charlotte Hallberg +46 13 28 16 57 ann-charlotte.hallberg@liu.se Annelie Almquist 013-28 29 34
Equipment permitted	Calculator. One handwritten A4 paper (both sides) with the students own notes.
Other important information	
Number of exams in the bag	

Linköpings Universitet
IDA/STIMA

732A93 Statistical Methods, 6hp

Exam in Statistical Methods, 2019-02-20

Time allowed: kl: 8-12

Allowed aids: Calculator. One handwritten A4 paper (both sides) with the students own notes.

Assisting teacher: Hector Rodriguez-Deniz

Grades: A=19-20 points, B=17-18p, C=14-16p, D=12-13p, E=10-11p, F=0-9

Provide a detailed report that shows motivation of the results.

1

Let $f(y) = cy^2,\ -1 \le y \le 1$ be the density function of the random variable Y.

a) Find c so that $f(y)$ is a density function. 1p

b) Find the cumulative distribution function $F(y)$. 1p

c) Calculate $P(Y \ge 1/2)$ and $P(-1/2 \le Y \le 1/2)$. 1p

d) Calculate $E[3Y + 10]$. 1p

2

Let the bivariate random variable (Y_1, Y_2) have joint density function:

$$f(y_1, y_2) = \begin{cases} (y_1 + y_2)/8000, & 0 \le y_1, y_2 \le 20 \\ 0, & elsewhere \end{cases}$$

a) Find the marginal distribution $f(y_2)$. 1p

b) Find the conditional distribution $P(Y_1 | Y_2 = 10)$. 1p

c) Calculate $P(Y_1 \le 14 | Y_2 = 10)$. 1p

d) Calculate $E(Y_1 | Y_2 = 10)$. 1p

3

Let $Y_1, Y_2, \ldots, Y_n$ denote n independent and identically distributed random variables from the following distribution with unknown parameter $\theta > 0$,

$$f(y|\theta) = \frac{2}{\theta} y e^{-y^2/\theta}, \qquad y > 0$$

$$E[Y] = \frac{\sqrt{\theta\pi}}{2}$$

$$V[Y] = \frac{\theta(4 - \pi)}{4}$$

a) Estimate θ using the Maximum Likelihood method. 3p

b) Show that $\hat{\theta}_{ML}$ is an unbiased estimator of θ. 1p

4

We collect the following random sample (yi,xi) of size n = 8 where Y is the dependent (output) variable and X the independent (input) variable.

Y	X
4	3
7	4
9	5
10	6
9	8
10	9
11	10
12	11

a) Set up the simple linear regression model and estimate β_0 and β_1. 2p

b) Test (independently) $H_0: \beta_0 = 0$ and $H_0: \beta_1 = 0$. You may use without showing any calculations that SSE= 10,25. Use 5% significance level in both cases. 2p

5

We want to estimate the number of patients arriving in an emergency room at Linköping's Hospital during 22:00-23:00 on Fridays. We collect a sample of size n, and we use a Poisson distribution for modeling the data, i.e. $Y \sim Poisson(\lambda)$. Now we place a Gamma prior on λ, i.e. $\lambda \sim Gamma(\alpha, \beta)$.

a) Find the posterior distribution of λ, including the proportionality constant. 2p

b) If we collect a sample such that $n = 10, \sum y = 71$, what would be your Bayes estimate for the number of patients after using a prior $Gamma(4,2)$?. 2p

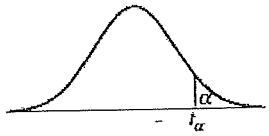
Discrete Distributions

Distribution	Probability Function	Mean	Variance
Binomial	$p(y) = \binom{n}{y} p^y (1-p)^{n-y};$ $y = 0, 1, \ldots, n$	np	$np(1-p)$
Geometric	$p(y) = p(1-p)^{y-1};$ $y = 1, 2, \ldots$	$\frac{1}{p}$	$\frac{1-p}{p^2}$
Hypergeometric	$p(y) = \frac{\binom{r}{y}\binom{N-r}{n-y}}{\binom{N}{n}};$ $y = 0, 1, \ldots, n$ if $n \leq r$, $y = 0, 1, \ldots, r$ if $n > r$	$\frac{nr}{N}$	$n\left(\frac{r}{N}\right)\left(\frac{N-r}{N}\right)\left(\frac{N-n}{N-1}\right)$
Poisson	$p(y) = \frac{\lambda^y e^{-\lambda}}{y!};$ $y = 0, 1, 2, \ldots$	λ	λ
Negative binomial	$p(y) = \binom{y-1}{r-1} p^r (1-p)^{y-r};$ $y = r, r+1, \ldots$	$\frac{r}{p}$	$\frac{r(1-p)}{p^2}$

Continuous Distributions

Distribution	Probability Function	Mean	Variance
Uniform	$f(y) = \frac{1}{\theta_2 - \theta_1}; \theta_1 \le y \le \theta_2$	$\frac{\theta_1 + \theta_2}{2}$	$\frac{(\theta_2 - \theta_1)^2}{12}$
Normal	$f(y) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\left(\frac{1}{2\sigma^2}\right)(y-\mu)^2\right]$ $-\infty < y < +\infty$	μ	σ^2
Exponential	$f(y) = \frac{1}{\beta} e^{-y/\beta}; \quad \beta > 0$ $0 < y < \infty$	β	β^2
Gamma	$f(y) = \left[\frac{1}{\Gamma(\alpha)\beta^\alpha}\right] y^{\alpha-1} e^{-y/\beta};$ $0 < y < \infty$	$\alpha\beta$	$\alpha\beta^2$
Chi-square	$f(y) = \frac{(y)^{(v/2)-1} e^{-y/2}}{2^{v/2}\Gamma(v/2)};$ $y^2 > 0$	v	$2v$
Beta	$f(y) = \left[\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\right] y^{\alpha-1}(1-y)^{\beta-1};$ $0 < y < 1$	$\frac{\alpha}{\alpha+\beta}$	$\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$

Table 5 Percentage Points of the *t* Distributions



$t_{.100}$	$t_{.050}$	$t_{.025}$	$t_{.010}$	$t_{.005}$	df
3.078	6.314	12.706	31.821	63.657	1
1.886	2.920	4.303	6.965	9.925	2
1.638	2.353	3.182	4.541	5.841	3
1.533	2.132	2.776	3.747	4.604	4
1.476	2.015	2.571	3.365	4.032	5
1.440	1.943	2.447	3.143	3.707	6
1.415	1.895	2.365	2.998	3.499	7
1.397	1.860	2.306	2.896	3.355	8
1.383	1.833	2.262	2.821	3.250	9
1.372	1.812	2.228	2.764	3.169	10
1.363	1.796	2.201	2.718	3.106	11
1.356	1.782	2.179	2.681	3.055	12
1.350	1.771	2.160	2.650	3.012	13
1.345	1.761	2.145	2.624	2.977	14
1.341	1.753	2.131	2.602	2.947	15
1.337	1.746	2.120	2.583	2.921	16
1.333	1.740	2.110	2.567	2.898	17
1.330	1.734	2.101	2.552	2.878	18
1.328	1.729	2.093	2.539	2.861	19
1.325	1.725	2.086	2.528	2.845	20
1.323	1.721	2.080	2.518	2.831	21
1.321	1.717	2.074	2.508	2.819	22
1.319	1.714	2.069	2.500	2.807	23
1.318	1.711	2.064	2.492	2.797	24
1.316	1.708	2.060	2.485	2.787	25
1.315	1.706	2.056	2.479	2.779	26
1.314	1.703	2.052	2.473	2.771	27
1.313	1.701	2.048	2.467	2.763	28
1.311	1.699	2.045	2.462	2.756	29
1.282	1.645	1.960	2.326	2.576	inf.