

Exam in Statistical Methods, 2018-11-26

Time allowed: kl: 8-12

Allowed aids: Calculator. One handwritten A4 paper (both sides) with the students own notes.

Assisting teacher: Hector Rodriguez-Deniz

Grades: A=19-20 points, B=17-18p, C=14-16p, D=12-13p, E=10-11p, F=0-9

Provide a detailed report that shows motivation of the results.

1

Let $f(y) = c(1 - y)$, $0 < y < 1$ be the density function of the random variable Y .

- a) Find c so that $f(y)$ is a density function. 1p
- b) Find the cumulative distribution function $F(y)$. 1p
- c) Find $E[Y]$. 1p
- d) Find $V[Y]$. 1p

2

Let the bivariate random variable (Y_1, Y_2) have joint density function:

$$f(y_1, y_2) = \begin{cases} 3y_1, & 0 \leq y_2 \leq y_1 \leq 1 \\ 0, & \text{elsewhere} \end{cases}$$

- a) Find the marginal distributions $f(y_1)$ and $f(y_2)$. Are y_1, y_2 independent? 2p
- b) Calculate $P(Y_1 \leq 1/2, Y_2 \leq 1/2)$. 1p
- c) Find $f(y_1|y_2)$ and use it to calculate $P(Y_1 \leq 3/4 | Y_2 = 1/2)$. 1p

3

Let Y_1, Y_2, \dots, Y_n denote n independent and identically distributed random variables from the following distribution with unknown parameter k ,

$$f(y|k) = \begin{cases} (k+1)y^k, & 0 < y < 1 \\ 0, & \text{elsewhere} \end{cases}$$

Estimate k using the Maximum Likelihood method. 3p

4

Assume that Y_1, Y_2 , and Y_3 are random variables, with

$$\begin{aligned} E(Y_1) &= 2 & V(Y_1) &= 6 & \text{Cov}(Y_1, Y_2) &= 1 \\ E(Y_2) &= -2 & V(Y_2) &= 4 & \text{Cov}(Y_1, Y_3) &= 0 \\ E(Y_3) &= 6 & V(Y_3) &= 1 & \text{Cov}(Y_2, Y_3) &= -1 \end{aligned}$$

- a) Find $E(Y_1 - 4Y_2 - Y_3)$. 1p
- b) Find $V(4Y_1 - 2Y_2 + Y_3)$. 1p

5

We collect the following random sample (y_i, x_i) of size $n = 7$ where Y is the dependent (output) variable and X the independent (input) variable.

Y	X
5	-2
4	-1
4	0
3	1
2	2
1	3
1	4

- a) Set up the simple linear regression model and estimate β_0 and β_1 . 2p
- b) Test if the slope is zero. You may use without showing any calculations that $SSE = 0,571$. Use 5% significance level. Interpret your result. 1p
- c) Find a 90% confidence interval for $E[Y] = \beta_0 + \beta_1 x^*$ at $x^* = 5$. 1p

6

Let Y_1, Y_2, \dots, Y_n denote a random sample from an exponentially distributed population with density $p(y|\theta) = \theta e^{-\theta y}$, $0 < y$, and unknown parameter $0 < \theta$.

- a) Choose an appropriate prior distribution for θ , and use it to find its posterior distribution. 2p
- b) Find an expression for the estimator $\hat{\theta}_{Bayes}$. 1p

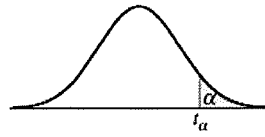
Discrete Distributions

Distribution	Probability Function	Mean	Variance
Binomial	$p(y) = \binom{n}{y} p^y (1-p)^{n-y};$ $y = 0, 1, \dots, n$	np	$np(1-p)$
Geometric	$p(y) = p(1-p)^{y-1};$ $y = 1, 2, \dots$	$\frac{1}{p}$	$\frac{1-p}{p^2}$
Hypergeometric	$p(y) = \frac{\binom{r}{y} \binom{N-r}{n-y}}{\binom{N}{n}};$ $y = 0, 1, \dots, n \text{ if } n \leq r,$ $y = 0, 1, \dots, r \text{ if } n > r$	$\frac{nr}{N}$	$n \left(\frac{r}{N} \right) \left(\frac{N-r}{N} \right) \left(\frac{N-n}{N-1} \right)$
Poisson	$p(y) = \frac{\lambda^y e^{-\lambda}}{y!};$ $y = 0, 1, 2, \dots$	λ	λ
Negative binomial	$p(y) = \binom{y-1}{r-1} p^r (1-p)^{y-r};$ $y = r, r+1, \dots$	$\frac{r}{p}$	$\frac{r(1-p)}{p^2}$

Continuous Distributions

Distribution	Probability Function	Mean	Variance
Uniform	$f(y) = \frac{1}{\theta_2 - \theta_1}; \theta_1 \leq y \leq \theta_2$	$\frac{\theta_1 + \theta_2}{2}$	$\frac{(\theta_2 - \theta_1)^2}{12}$
Normal	$f(y) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\left(\frac{1}{2\sigma^2}\right)(y - \mu)^2\right]$ $-\infty < y < +\infty$	μ	σ^2
Exponential	$f(y) = \frac{1}{\beta} e^{-y/\beta}; \beta > 0$ $0 < y < \infty$	β	β^2
Gamma	$f(y) = \left[\frac{1}{\Gamma(\alpha)\beta^\alpha}\right] y^{\alpha-1} e^{-y/\beta};$ $0 < y < \infty$	$\alpha\beta$	$\alpha\beta^2$
Chi-square	$f(y) = \frac{(y)^{(v/2)-1} e^{-y/2}}{2^{v/2}\Gamma(v/2)};$ $y^2 > 0$	v	$2v$
Beta	$f(y) = \left[\frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)}\right] y^{\alpha-1} (1 - y)^{\beta-1};$ $0 < y < 1$	$\frac{\alpha}{\alpha + \beta}$	$\frac{\alpha\beta}{(\alpha + \beta)^2(\alpha + \beta + 1)}$

Table 5 Percentage Points of the t Distributions



$t_{.100}$	$t_{.050}$	$t_{.025}$	$t_{.010}$	$t_{.005}$	df
3.078	6.314	12.706	31.821	63.657	1
1.886	2.920	4.303	6.965	9.925	2
1.638	2.353	3.182	4.541	5.841	3
1.533	2.132	2.776	3.747	4.604	4
1.476	2.015	2.571	3.365	4.032	5
1.440	1.943	2.447	3.143	3.707	6
1.415	1.895	2.365	2.998	3.499	7
1.397	1.860	2.306	2.896	3.355	8
1.383	1.833	2.262	2.821	3.250	9
1.372	1.812	2.228	2.764	3.169	10
1.363	1.796	2.201	2.718	3.106	11
1.356	1.782	2.179	2.681	3.055	12
1.350	1.771	2.160	2.650	3.012	13
1.345	1.761	2.145	2.624	2.977	14
1.341	1.753	2.131	2.602	2.947	15
1.337	1.746	2.120	2.583	2.921	16
1.333	1.740	2.110	2.567	2.898	17
1.330	1.734	2.101	2.552	2.878	18
1.328	1.729	2.093	2.539	2.861	19
1.325	1.725	2.086	2.528	2.845	20
1.323	1.721	2.080	2.518	2.831	21
1.321	1.717	2.074	2.508	2.819	22
1.319	1.714	2.069	2.500	2.807	23
1.318	1.711	2.064	2.492	2.797	24
1.316	1.708	2.060	2.485	2.787	25
1.315	1.706	2.056	2.479	2.779	26
1.314	1.703	2.052	2.473	2.771	27
1.313	1.701	2.048	2.467	2.763	28
1.311	1.699	2.045	2.462	2.756	29
1.282	1.645	1.960	2.326	2.576	inf.