

Exam in Statistical Methods, 2018-02-19

Time allowed: kl: 8-12

Allowed aids: Calculator. One handwritten A4 paper (both sides) with the students own notes.

Assisting teacher: Hector Rodriguez-Deniz

Grades: A=19-20 points, B=17-18p, C=14-16p, D=12-13p, E=10-11p, F=0-9

Provide a detailed report that shows motivation of the results.

1

Let $f(y) = c(1-y),\ 0 \leq y \leq 1$ be the density function of the random variable Y.

a) Find c so that $f(y)$ is a density function. 1p

b) Find the cumulative distribution function $F(y)$. 1p

c) Find $E[Y]$. 1p

d) Find $V[Y]$. 1p

2

Let the bivariate random variable (Y_1, Y_2) have joint density function:

$$f(y_1, y_2) = \begin{cases} 3y_1, & 0 \leq y_2 \leq y_1 \leq 1 \\ 0, & elsewhere \end{cases}$$

a) Find the marginal distributions $f(y_1)$ and $f(y_2)$. 2p

b) Calculate $P(Y_1 \leq 1/2\ , Y_2 \leq 1/2)$. 1p

c) Find $f(y_1|y_2)$ and use it to calculate $P(Y_1 \leq 3/4\ |Y_2 = 1/2)$. 1p

3

Let $Y_1, Y_2, \ldots, Y_n$ denote n independent and identically distributed random variables from the following distribution with unknown parameter k,

$$f(y|k) = \begin{cases} (k+1)y^k, & 0 \leq y \leq 1 \\ 0, & elsewhere \end{cases}$$

a) Estimate k using the Maximum Likelihood method. 3p

4

Assume that Y_1, Y_2, and Y_3 are random variables, with

$$E(Y_1) = 2 \quad V(Y_1) = 4 \quad Cov(Y_1, Y_2) = 1$$
$$E(Y_2) = -2 \quad V(Y_2) = 2 \quad Cov(Y_1, Y_3) = 0$$
$$E(Y_3) = 6 \quad V(Y_3) = 8 \quad Cov(Y_2, Y_3) = -1$$

a) Find $E(2Y_1 + 4Y_2 - 4Y_3)$. 1p

b) Find $V(4Y_1 - 2Y_2 + Y_3)$. 1p

5

The following data on energy consumption was collected by Tekniska verken in Linköping:

Y	X
70	0
57	8
60	7.5
63	13.5
57	14
67	4.5
107	-11

Where

Y = Average daily energy consumption (kW)
X = Average daily temperature (C)

a) Set up the simple linear regression model and estimate the parameters β_0 , β_1 2p

b) Test if β_1 is zero. You may use without showing any calculations that SSE = 324,02. Use 5% significance level. Interpret your result. 2p

6

Let $Y_1, Y_2, \ldots, Y_n$ denote a random sample from an exponentially distributed population with density $p(y|\theta) = \theta e^{-\theta y}, 0 < y,$ and unknown parameter $0 < \theta$.

a) Choose an appropriate prior distribution for θ, and use it to find its posterior distribution.

2p

b) Find an expression for the estimator $\hat{\theta}_{Bayes}$. 1p

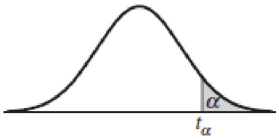
Discrete Distributions

Distribution	Probability Function	Mean	Variance
Binomial	$p(y) = \binom{n}{y} p^y (1-p)^{n-y};$ $y = 0, 1, \ldots, n$	np	$np(1-p)$
Geometric	$p(y) = p(1-p)^{y-1};$ $y = 1, 2, \ldots$	$\frac{1}{p}$	$\frac{1-p}{p^2}$
Hypergeometric	$p(y) = \frac{\binom{r}{y}\binom{N-r}{n-y}}{\binom{N}{n}};$ $y = 0, 1, \ldots, n$ if $n \le r$, $y = 0, 1, \ldots, r$ if $n > r$	$\frac{nr}{N}$	$n\left(\frac{r}{N}\right)\left(\frac{N-r}{N}\right)\left(\frac{N-n}{N-1}\right)$
Poisson	$p(y) = \frac{\lambda^y e^{-\lambda}}{y!};$ $y = 0, 1, 2, \ldots$	λ	λ
Negative binomial	$p(y) = \binom{y-1}{r-1} p^r (1-p)^{y-r};$ $y = r, r+1, \ldots$	$\frac{r}{p}$	$\frac{r(1-p)}{p^2}$

Continuous Distributions

Distribution	Probability Function	Mean	Variance
Uniform	$f(y) = \frac{1}{\theta_2 - \theta_1}; \theta_1 \le y \le \theta_2$	$\frac{\theta_1 + \theta_2}{2}$	$\frac{(\theta_2 - \theta_1)^2}{12}$
Normal	$f(y) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\left(\frac{1}{2\sigma^2}\right)(y-\mu)^2\right]$ $-\infty < y < +\infty$	μ	σ^2
Exponential	$f(y) = \frac{1}{\beta} e^{-y/\beta}; \quad \beta > 0$ $0 < y < \infty$	β	β^2
Gamma	$f(y) = \left[\frac{1}{\Gamma(\alpha)\beta^\alpha}\right] y^{\alpha-1} e^{-y/\beta};$ $0 < y < \infty$	$\alpha\beta$	$\alpha\beta^2$
Chi-square	$f(y) = \frac{(y)^{(v/2)-1} e^{-y/2}}{2^{v/2}\Gamma(v/2)};$ $y^2 > 0$	v	$2v$
Beta	$f(y) = \left[\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\right] y^{\alpha-1}(1-y)^{\beta-1};$ $0 < y < 1$	$\frac{\alpha}{\alpha+\beta}$	$\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$

Table 5 Percentage Points of the t Distributions



$t_{.100}$	$t_{.050}$	$t_{.025}$	$t_{.010}$	$t_{.005}$	df
3.078	6.314	12.706	31.821	63.657	1
1.886	2.920	4.303	6.965	9.925	2
1.638	2.353	3.182	4.541	5.841	3
1.533	2.132	2.776	3.747	4.604	4
1.476	2.015	2.571	3.365	4.032	5
1.440	1.943	2.447	3.143	3.707	6
1.415	1.895	2.365	2.998	3.499	7
1.397	1.860	2.306	2.896	3.355	8
1.383	1.833	2.262	2.821	3.250	9
1.372	1.812	2.228	2.764	3.169	10
1.363	1.796	2.201	2.718	3.106	11
1.356	1.782	2.179	2.681	3.055	12
1.350	1.771	2.160	2.650	3.012	13
1.345	1.761	2.145	2.624	2.977	14
1.341	1.753	2.131	2.602	2.947	15
1.337	1.746	2.120	2.583	2.921	16
1.333	1.740	2.110	2.567	2.898	17
1.330	1.734	2.101	2.552	2.878	18
1.328	1.729	2.093	2.539	2.861	19
1.325	1.725	2.086	2.528	2.845	20
1.323	1.721	2.080	2.518	2.831	21
1.321	1.717	2.074	2.508	2.819	22
1.319	1.714	2.069	2.500	2.807	23
1.318	1.711	2.064	2.492	2.797	24
1.316	1.708	2.060	2.485	2.787	25
1.315	1.706	2.056	2.479	2.779	26
1.314	1.703	2.052	2.473	2.771	27
1.313	1.701	2.048	2.467	2.763	28
1.311	1.699	2.045	2.462	2.756	29
1.282	1.645	1.960	2.326	2.576	inf.