

Information page for written examinations at Linköping University



Examination date	2016-11-21
Room (1)	<u>TER3</u>
Time	8-12
Course code	732A49
Exam code	TENT
Course name Exam name	Statistical Methods (Statistiska metoder) Examination (Tentamen)
Department	IDA
Number of questions in the examination	4
Teacher responsible/contact person during the exam time	Lotta Hallberg
Contact number during the exam time	013-281657
Visit to the examination room approximately	10
Name and contact details to the course administrator (name + phone nr + mail)	Annelie Almquist
Equipment permitted	Calculator. One handwritten A4 paper (both sides) with the students own notes.
Other important information	
Number of exams in the bag	

Exam in Statistical Methods, 2016-11-21

Time allowed: kl: 8-12
Allowed aids: Calculator. One handwritten A4 paper (both sides) with the students own notes.
Assisting teacher: Lotta Hallberg
Grades: A=19-20 points, B=17-18p, C=14-16p, D=12-13p, E=10-11p
Provide a detailed report that shows motivation of the results.

1

The management of a newspaper wonders whether stockpiles decline may be due to stressed staff. It makes an investigation along with a psychologist and finds that of the $n = 40$ employees who are included in the survey, so feel $x = 8$ people stressed.

Estimate p = the proportion stressed at the newspaper using:

- a) ML-method 2p
- b) Method of moments 1p
- c) Bayes method. Use $beta(2,4)$ as prior distribution. 3p
- d) Show that the estimator $\hat{p} = \frac{x}{n}$ is an unbiased and consistent estimator of p . 2p

If we want to investigate if the proportion stressed is larger than 10% we use the hypotheses $H_0: p = 10\%$ against $H_a: p > 10\%$.

- e) Calculate the approximate p-value for this test using the observation above. 2p
- f) If we would like to calculate an approximate confidence interval for p . which pivotal quantity is appropriate to use? 1p

2

Boxes are packed with 1 packet of sugar, 2 packets of flour, 10 packets of cereal and 6 packets of oatmeal. Let the random variables have distributions:

$$X_1 = \text{weight of a sugar packet} \sim N(2kg, 0.3kg)$$

$$X_2 = \text{weight of a flour packet} \sim N(2kg, 0.1kg)$$

$$X_3 = \text{weight of a cereal packet} \sim N(750g, 50g)$$

$$X_4 = \text{weight of a oatmeal packet} \sim N(500g, 10g)$$

Box weight is negligible and the random variables above are of course independent and the weight of all 19 packets are independent.

If a box is selected at random what is the probability that the box and its contents weigh more than 17 kg? 3p

3

Let the random variable X have density function $f(x) = \frac{x}{50}$ when $0 < x < 10$. Elsewhere the density is 0.

Calculate:

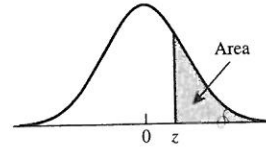
- a) The expected value of X 1.5p
- b) The median of X 1.5p

4

The density function for the proportion X is given by $f(x|\theta) = \theta x^{\theta-1}$ when $0 < x < 1$, $\theta > 0$, and $f(x) = 0$ elsewhere.

Let $Y = -\ln X$. Show that $Y \sim \exp(\theta)$ (the exponential distribution). 3p

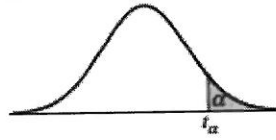
Table 4 Normal Curve Areas
Standard normal probability in right-hand tail
 (for negative values of z , areas are found by symmetry)



z	Second decimal place of z									
	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641
0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0722	.0708	.0694	.0681
1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
1.8	.0359	.0352	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
2.9	.0019	.0018	.0017	.0017	.0016	.0016	.0015	.0015	.0014	.0014
3.0	.00135									
3.5	.000233									
4.0	.0000317									
4.5	.00000340									
5.0	.000000287									

From R. E. Walpole, *Introduction to Statistics* (New York: Macmillan, 1968).

Percentage Points of the t Distributions



$t_{.100}$	$t_{.050}$	$t_{.025}$	$t_{.010}$	$t_{.005}$	df
3.078	6.314	12.706	31.821	63.657	1
1.886	2.920	4.303	6.965	9.925	2
1.638	2.353	3.182	4.541	5.841	3
1.533	2.132	2.776	3.747	4.604	4
1.476	2.015	2.571	3.365	4.032	5
1.440	1.943	2.447	3.143	3.707	6
1.415	1.895	2.365	2.998	3.499	7
1.397	1.860	2.306	2.896	3.355	8
1.383	1.833	2.262	2.821	3.250	9
1.372	1.812	2.228	2.764	3.169	10
1.363	1.796	2.201	2.718	3.106	11
1.356	1.782	2.179	2.681	3.055	12
1.350	1.771	2.160	2.650	3.012	13
1.345	1.761	2.145	2.624	2.977	14
1.341	1.753	2.131	2.602	2.947	15
1.337	1.746	2.120	2.583	2.921	16
1.333	1.740	2.110	2.567	2.898	17
1.330	1.734	2.101	2.552	2.878	18
1.328	1.729	2.093	2.539	2.861	19
1.325	1.725	2.086	2.528	2.845	20
1.323	1.721	2.080	2.518	2.831	21
1.321	1.717	2.074	2.508	2.819	22
1.319	1.714	2.069	2.500	2.807	23
1.318	1.711	2.064	2.492	2.797	24
1.316	1.708	2.060	2.485	2.787	25
1.315	1.706	2.056	2.479	2.779	26
1.314	1.703	2.052	2.473	2.771	27
1.313	1.701	2.048	2.467	2.763	28
1.311	1.699	2.045	2.462	2.756	29
1.282	1.645	1.960	2.326	2.576	inf.

Table 1 Discrete Distributions

Distribution	Probability Function	Mean	Variance	Moment-Generating Function
Binomial	$p(y) = \binom{n}{y} p^y (1-p)^{n-y};$ $y = 0, 1, \dots, n$	np	$np(1-p)$	$[pe^t + (1-p)]^n$
Geometric	$p(y) = p(1-p)^{y-1};$ $y = 1, 2, \dots$	$\frac{1}{p}$	$\frac{1-p}{p^2}$	$\frac{pe^t}{1-(1-p)e^t}$
Hypergeometric	$p(y) = \frac{\binom{r}{y} \binom{N-r}{n-y}}{\binom{N}{n}};$ $y = 0, 1, \dots, n$ if $n \leq r,$ $y = 0, 1, \dots, r$ if $n > r$	$\frac{nr}{N}$	$n \left(\frac{r}{N} \right) \left(\frac{N-r}{N} \right) \left(\frac{N-n}{N-1} \right)$	does not exist in closed form
Poisson	$p(y) = \frac{\lambda^y e^{-\lambda}}{y!};$ $y = 0, 1, 2, \dots$	λ	λ	$\exp[\lambda(e^t - 1)]$
Negative binomial	$p(y) = \binom{y-1}{r-1} p^r (1-p)^{y-r};$ $y = r, r+1, \dots$	$\frac{r}{p}$	$\frac{r(1-p)}{p^2}$	$\left[\frac{pe^t}{1-(1-p)e^t} \right]^r$

Table 2 Continuous Distributions

Distribution	Probability Function	Mean	Variance	Moment-Generating Function
Uniform	$f(y) = \frac{1}{\theta_2 - \theta_1}; \theta_1 \leq y \leq \theta_2$	$\frac{\theta_1 + \theta_2}{2}$	$\frac{(\theta_2 - \theta_1)^2}{12}$	$\frac{e^{t\theta_2} - e^{t\theta_1}}{t(\theta_2 - \theta_1)}$
Normal	$f(y) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\left(\frac{1}{2\sigma^2}\right)(y - \mu)^2\right]$ $-\infty < y < +\infty$	μ	σ^2	$\exp\left(\mu t + \frac{t^2\sigma^2}{2}\right)$
Exponential	$f(y) = \frac{1}{\beta} e^{-y/\beta}; \beta > 0$ $0 < y < \infty$	β	β^2	$(1 - \beta t)^{-1}$
Gamma	$f(y) = \left[\frac{1}{\Gamma(\alpha)\beta^\alpha}\right] y^{\alpha-1} e^{-y/\beta};$ $0 < y < \infty$	$\alpha\beta$	$\alpha\beta^2$	$(1 - \beta t)^{-\alpha}$
Chi-square	$f(y) = \frac{(y)^{(v/2)-1} e^{-y/2}}{2^{v/2}\Gamma(v/2)};$ $y > 0$	v	$2v$	$(1 - 2t)^{-v/2}$
Beta	$f(y) = \left[\frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)}\right] y^{\alpha-1}(1 - y)^{\beta-1};$ $0 < y < 1$	$\frac{\alpha}{\alpha + \beta}$	$\frac{\alpha\beta}{(\alpha + \beta)^2(\alpha + \beta + 1)}$	does not exist in closed form

