Examination Probability Theory

Linköpings Universitet, IDA, Statistik

Course code and name:	732A63 Probability Theory
Date:	2017/11/30, 8–12
Examinator:	Krzysztof Bartoszek phone 013–281 885
Allowed aids:	Pocket calculator
	Table with common formulae and moment generating functions (distributed with the exam)
	Table of integrals (distributed with the exam)
	Table with distributions from Appendix B in the course book (distributed with the exam)
Grades:	A= $[19-20]$ points
	B= $[17-19)$ points
	C= $[12-17)$ points
	D= $[10-12)$ points
	E= $[8-10)$ points
	F= $[0-8)$ points **(FAIL)**
Instructions:	Write clear and concise answers to the questions.
	Make sure to specify the support region for all density functions.

Problem 1 (5p)

Let X be continuous with density function $f(x) = C(x - x^2)$, where $\alpha < x < \beta$ and $C > 0$.

a) What are the possible values of α and β?

b) What can C be?

Problem 2 (5p)

Show that if X and Y are independent $\mathcal{N}(0, 1)$—distributed random variables, then $X/Y \sim$Cauchy$(0, 1)$.

Problem 3 (5p)

Show by using moment generating functions that if $X \sim$Laplace(1), then $X \overset{\mathcal{D}}{=} Y_1 - Y_2$, where Y_1 and Y_2 are independent, exponentially distributed random variables.

Problem 4 (5p)

Let $Y_1, Y_2, \ldots, Y_n$ be independent identically distributed random variables, each of which can take any value in the set of integers $\{0, 1, \ldots, 9\}$ with equal probability 1/10. Let

$$X_n = \sum_{i=1}^{n} Y_i 10^{-i}.$$

Show by the use of characteristic functions that X_n converges in distribution to Unif[0, 1].
TIP: The characteristic function of a discrete uniform distribution on the integer set $\{0, 1, \ldots, n\}$ is

$$\phi(t) = \frac{1}{n+1} \frac{1 - e^{(n+1)it}}{1 - e^{it}}$$

TIP: For any real, both positive x and y we have

$$\lim_{x \to \infty} x(1 - e^{iy/x}) = -iy.$$