Examination Probability Theory

Linköpings Universitet, IDA, Statistik

Course code and name:	732A63 Probability Theory
Date:	2017/10/23, 8–12
Examinator:	Krzysztof Bartoszek phone 013–281 885
Allowed aids:	Pocket calculator
	Table with common formulae and moment generating functions (distributed with the exam)
	Table of integrals (distributed with the exam)
	Table with distributions from Appendix B in the course book (distributed with the exam)
Grades:	A= $[19-\infty)$ points
	B= $[17-19)$ points
	C= $[12-17)$ points
	D= $[10-12)$ points
	E= $[8-10)$ points
	F= $[0-8)$ points **(FAIL)**
Instructions:	Write clear and concise answers to the questions.
	Make sure to specify the support region for all density functions.

Problem 1 (4p)

Show that if $X \sim Cauchy(0,1)$, then so is $1/X$.

Problem 2 (5p)

The random variables X and Y have a joint probability density of the form

$$f_{X,Y}(x,y) = \begin{cases} a(x+y)^2 & \text{if } 0<y<x<1 \\ 0 & otherwise \end{cases}$$

(a 1p) Are X and Y independent?
(b 1p) Determine the constant a.
(c 1p) Compute the marginal density of Y
(d 1p) What is the marginal density of X and why?
(e 1p) Compute the conditional density of $X|Y=y$.

Problem 3 (6p)

Assume that you take a bus every working day to University. You arrive every day at the bus stop and wait for an random time that is exponentially distributed with rate $10(\text{min})^{-1}$. Assume you go to University 200 times in the year.
(a 2p) What is the distribution of your minimal waiting time?
(b 2p) What can you say about the distribution of your maximal waiting time?
(c 2p) Now from the new year the bus company introduced a new scheduling system and your waiting time that changed to be exponentially distributed with rate $20(\text{min})^{-1}$. Do you expect that you will wait longer or shorter for the bus? What is the distribution of the minimum of your waiting time over two years (400 days— the previous year with the old system and the new year with the new system)?

Problem 4 (5p)

Suppose that $X_1, X_2, \ldots$ are independent, Pareto$(1, 2)$–distributed random variables and set $Y_n = \min\{X_1, X_2, \ldots, X_n\}$.
(a 2p) Show that $Y_n \overline{P}\!\!\rightarrow 1$ as $n \to \infty$. It thus follows that $Y_n \approx 1$ with a probability close to 1 when n is large. One might therefore suspect that there exists a limit theorem to the effect that $Y_n - 1$, suitably rescaled, converges in distribution as $n \to \infty$ (note that $Y_n > 1$ always).
(b 3p) Show that $n(Y_n - 1)$ converges in distribution as $n \to \infty$, and determine the limit distribution.