LINKÖPINGS UNIVERSITET
Institutionen för datavetenskap
Avdelningen för statistik
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2016-10-18
Probability Theory, 6 hp
732A63

Exam in Probability Theory, 6 credits

Exam time:	8-12
Allowed:	Pocket calculator. Table with common formulas and moment generating functions (distributed with the exam). Table of integrals (distributed with the exam). Table with distributions from Appendix B in the course book (distributed with the exam).
Examinator:	Mattias Villani.
Assisting teacher:	Per Sidén, phone *0704-977175*
Grades:	Grades: Maximum is 20 points. A: 19 points B: 17 points C: 14 points D: 12 points E: 10 points F: <10 points

- Write clear and concise answers to the questions.
- Make sure to specify the definition region for all density functions.

1. Assume $X \sim N(0,1)$ and $Y \sim N(0,4)$ and that X and Y are independent.
   (a) Compute $E[(X+1)(Y-1)]$. 1p.
   (b) Compute $E[Y|X+Y=6]$. 1.5p.
   (c) Derive the distribution of $U = \frac{X}{Y}$. Does it belong to a known distribution? 2.5p.

2. Let $Y|\theta \sim Bin(n,\theta)$, where n is a known positive integer. Let the density of θ be

   $$f_\theta(\theta) = a \cdot \theta^3$$

   for $\theta \in [0,1]$ and $f_\theta(\theta) = 0$ otherwise.
   (a) Determine the constant a, so that $f_\theta(\theta)$ is a proper density. 1p.
   (b) Calculate the variance of Y. 2p.
   (c) Compute the density of Y. 2p.

3. Consider a fair die with probability 1/6 of rolling a six. Consider a game where the die is rolled until a six comes up and let the random variable X denote the number of die rolls required until this event happens.

   (a) Determine the probability function of X and state if it belongs to a known distribution. 1p.

   (b) Assume that the same game is played n times and let X_{max} be the maximum value of X across all games. Derive the distribution function of X_{max}. 2p.

   (c) How many times does one have to play the game for having at least 50% chance of obtaining a value of X_{max} that is greater than or equal to 30? 2p.

4. Let X and Y be random variables such that

$$Y|X = x \sim N(0, x)$$

   with $X \sim Po(\lambda)$.

   (a) Find the characteristic function of Y. 1p.

   (b) Show that

$$\frac{Y}{\sqrt{\lambda}} \xrightarrow{d} N(0, 1)$$

   as $\lambda \to \infty$. 2p.

   (c) Formulate and prove the Central Limit Theorm. 2p.

Good luck!

Per