Examination

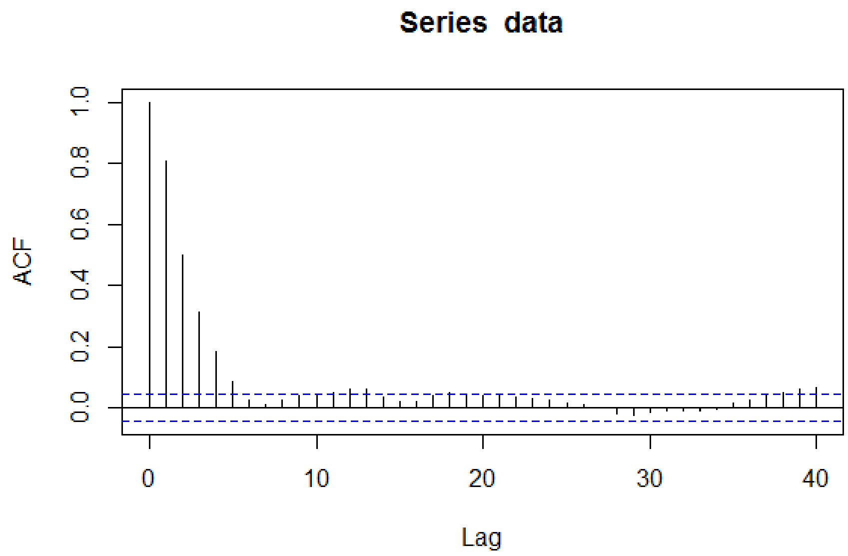
Linköping University, Department of Computer and Information Science, Statistics and Machine Learning

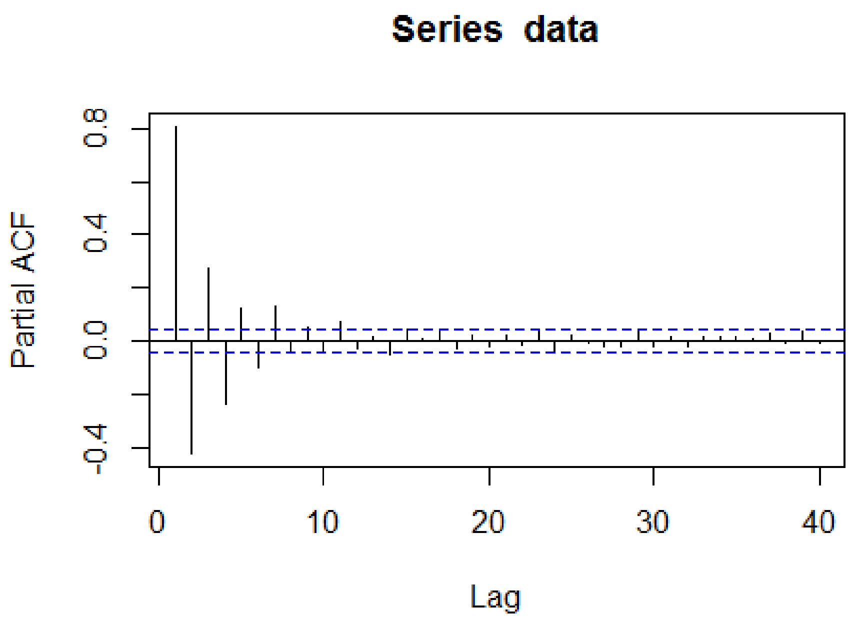
Course code and name	732A62 Time Series Analysis
Date and time	2017-10-26, 08.00-12.00
Assisting teacher	Oleg Sysoev
Allowed aids	"Time series analysis and its application" by Shumway & Stoffer or/and "Time series analysis" by Cryer and Chan, Information Sheet, Calculator.
Grades:	A=19-20 points B=16-18 points C=13-15 points D=11-12 points E=9-10 points F=0-8 points

Give motivated answers to the questions. If an answer is not motivated, the points are reduced.

1. For time series $X = (x_1, .. x_5) = (2,4,-2,6,10)$, compute a sample autocorrelation $\hat{\rho}_2$ **(2p)**
2. Assume that x_t and y_t are two uncorrelated AR(1) processes with mean zero, $\sigma_w^2 = 1$, and $\phi = 0.2$ for process x_t and $\phi = 0.5$ for process y_t. Consider $z_t = x_t + 0.3y_t$. Is z_t stationary? Compute its autocovariance function **(2p)**
3. Determine ARMA orders for a stationary process $x_t = 2 - 0.5x_{t-1} + w_t + 0.1w_{t-1}$. Use this information to compute ACF values for this process (you don't need to derive ACF formula yourself). Make also a long-range forecast, i.e. find out the limiting prediction value $\tilde{x}_{n+m}$ when $m \to \infty$. **(2p)**
4. Check the invertibility and causality of the following process and determine its orders in $ARIMA(p,d,q) \times (P,D,Q)_s$ notation **(3p)**:
   a. $x_t = x_{t-1} + 2x_{t-4} - 2x_{t-5} + w_t - 0.3w_{t-1} - 0.1w_{t-2}$
5. Estimate coefficients of the AR(2) process using Yule-Walker equations by using information about the following autocorrelations: $\hat{\rho}(1) = 0.6$, $\hat{\rho}(2) = 0.2$ and write down the estimated model equation.**(2p)**

6. For the AR(2) model with estimated parameters $\phi_1 = 0.8$, $\phi_2 = -0.5, \mu = 5$ and $\sigma_w^2 = 1$, compute a 95% prediction interval for the two-step-ahead forecast, i.e. the interval for $\tilde{x}_{n+2}^n$. Assume that $x_n = 6$, $x_{n-1} = 8$. **(3p)**
7. Write down an equation of $ARIMA(0,1,1) \times (2,0,0)_6$ and determine which $ARIMA(p, d, q)$ model it may correspond to. **(1p)**
8. Look at the figures below and determine which ARIMA model of the following models is the most suitable: ARIMA(0,0,6), ARIMA(1,0,1), ARIMA(7,0,5). Motivate your answer. **(1p)**





9. Look at the figure below and determine which GARCH models of the following models is the most suitable: GARCH(1,2), GARCH(2,1), GARCH(2,2) . Motivate your answer. **(1p)**

```
> TSA::eacf(data^2)
AR/MA
  0 1 2 3 4 5 6 7 8 9 10 11 12 13
0 x x x x o o o o o o o  o  o  o
1 x x o o o o o o o o o  o  o  o
2 x o o o o o o o o o o  o  o  o
3 x x o o o o o o o o o  o  o  o
4 x x x o o o o o o o o  o  o  o
5 x x x x o o o o o o o  o  o  o
6 x x o x o o o o o o o  o  o  o
7 x x o x x x o o o o o  o  o  o
```

10. After applying a seasonal differencing with period $s = 12$ to data series x_t, an ARIMA model was fit to the resulting data $y_t = \nabla_{12} x_t$ and the following output was obtained. By using this output, write down the equation of the estimated model in terms of x_t and investigate whether the model is reducible or not. Present the final model equation (if reducible, fix this problem first). The final equation should not contain backshift operators. **(2p)**

```
> arima(data, order = c(1,0,2))

Call:
arima(x = data, order = c(1, 0, 2))

Coefficients:
          ar1     ma1      ma2  intercept
      -0.6822  0.1657  -0.5784    -0.0304
se.    0.1725  0.1576   0.0988     0.0321
```

11. Determine the sine and the cosine amplitudes of the frequency $w = 0.1$ by knowing that $n = 100$ and looking at the following output of the Discrete Fourier Transform $d\left(\frac{j}{n}\right)$ **(1p)**:

```
> d_w
Time Series:
Start = 1
End = 100
Frequency = 1
  [1] -0.3178445+0.0000000i -0.3589806+0.0755837i -0.1892396+0.1945842i  0.2029804-0.2869672i  0.5454744+0.2230626i
  [6] -0.2120019+0.3755299i -0.2033392+0.0213895i  0.0997303-0.3245039i  0.3471806-0.0049712i -0.1613730+0.7358558i
 [11] -0.2628591-0.1884668i  0.4755845-0.1637641i -0.2228794-0.4961815i  0.1699727-0.6155358i  0.6705536+0.4506432i
 [16]  0.1878075+0.7227206i  0.0312714-0.1132900i -0.2683178+1.4487834i -0.8189616-0.6629312i  0.6612961-0.5629343i
 [21] -0.2272997-0.3131199i -0.4083954+0.2459699i  1.0130480+0.4680813i -0.9567709+1.5908104i -0.9626990-0.8586150i
 [26]  1.0216143+0.9699860i  0.8894355+0.9648221i  0.4177923-0.5638944i -0.2176846+1.6146570i  0.4500253-0.7771727i
 [31] -0.7709906+0.4876455i -0.6182064-0.7530244i  0.1262764+0.7975134i  0.7088413+0.2817067i -0.0451891+1.0683996i
 [36]  1.2293375-0.8427702i  0.3241282+0.1837064i -3.0180123-1.1145472i -0.6914161-0.6160959i  0.6943809+1.4476735i
 [41]  1.8728691-0.7850129i -0.3529247+0.9450142i  1.1730722-1.5984589i -0.3468271-1.0955862i  0.8983568+0.4662211i
 [46]  0.0604524+0.6418081i  0.8262510-0.6674243i -1.0350619+0.6761142i -0.8742387+0.3526892i -0.2912167+0.6240876i
 [51] -1.5086348-0.0000000i -0.2912167-0.6240876i -0.8742387-0.3526892i -1.0350619-0.6761142i  0.8262510+0.6674243i
 [56]  0.0604524-0.6418081i  0.8983568-0.4662211i -0.3468271+1.0955862i  1.1730722+1.5984589i -0.3529247-0.9450142i
 [61]  1.8728691+0.7850129i  0.6943809-1.4476735i -0.6914161+0.6160959i -3.0180123+1.1145472i  0.3241282-0.1837064i
 [66]  1.2293375+0.8427702i -0.0451891-1.0683996i  0.7088413-0.2817067i  0.1262764-0.7975134i -0.6182064+0.7530244i
 [71] -0.7709906-0.4876455i  0.4500253+0.7771727i -0.2176846-1.6146570i  0.4177923+0.5638944i  0.8894355-0.9648221i
 [76]  1.0216143-0.9699860i -0.9626990+0.8586150i -0.9567709-1.5908104i  1.0130480-0.4680813i -0.4083954-0.2459699i
 [81] -0.2272997+0.3131199i  0.6612961+0.5629343i -0.8189616+0.6629312i -0.2683178-1.4487834i  0.0312714+0.1132900i
 [86]  0.1878075-0.7227206i  0.6705536-0.4506432i  0.1699727+0.6155358i -0.2228794+0.4961815i  0.4755845+0.1637641i
 [91] -0.2628591+0.1884668i -0.1613730-0.7358558i  0.3471806+0.0049712i  0.0997303+0.3245039i -0.2033392-0.0213895i
 [96] -0.2120019-0.3755299i  0.5454744-0.2230626i  0.2029804+0.2869672i -0.1892396-0.1945842i -0.3589806-0.0755837i
```