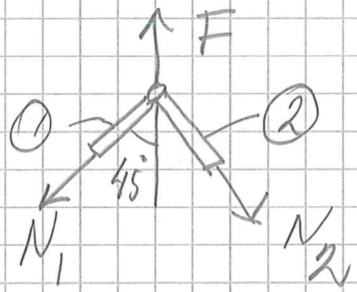
Hållfasthetslära Z2 TMEO17 2014-08-21

1) Se LB kap 4. Inför stångkrafter ställ upp jämvikt för knutpunkter

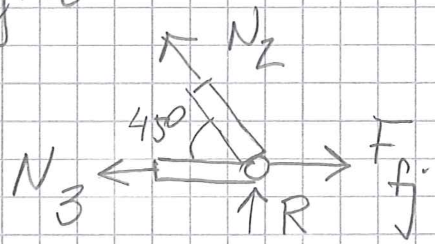
övre:



$$\rightarrow \quad \frac{N_2}{\sqrt{2}} - \frac{N_1}{\sqrt{2}} = 0 \Rightarrow N_1 = N_2$$

$$\uparrow \quad F - \frac{N_1}{\sqrt{2}} - \frac{N_2}{\sqrt{2}} \Rightarrow N_2 = N_1 = \frac{F}{\sqrt{2}} \quad (1)$$

nedre högra



$$\rightarrow \quad F_{fj} - N_3 - N_1/\sqrt{2} = 0 \quad (2)$$

Knäckning i stång 3 (horisontell)
Ledad infästning, LB p124, 126-127
Euler fall 2

$$P_{kr} = -N_3 = \frac{\pi^2 EI}{L^2} = \left\{ I = \frac{\pi d^4}{64} \right\} = \frac{\pi^3 E d^4}{64 L^2}$$

Hållfasthetslära Z2 TME017 2014-08-21

1) Forts:

Stång 3:s förlängning, LB P 11,

$$\delta_3 = \frac{N_3 L}{EA} = -\frac{\pi^3 E d^4}{64 L^2 E} \cdot \frac{4}{\pi d^2} = -\frac{\pi^2 d^2}{16 L}$$

Fjäderkraft $F_{fj} = k(-\delta_3) = \frac{\pi d^2 E}{40 L} \cdot \frac{\pi^2 d^2}{16 L}$

2) ⇒ $N_1 = \sqrt{2}\,(F_{fj} - N_3)$ med 1)

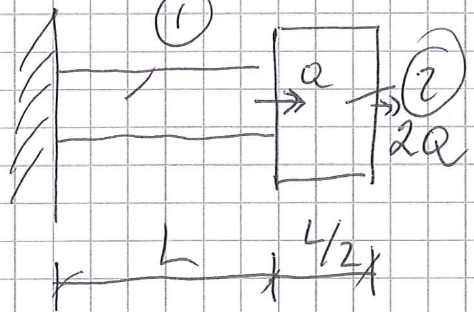
$$F = 2F_{fj} - 2N_3 =$$

$$= \frac{2\pi d^2 E}{40 L} \cdot \frac{\pi^2 d^2}{16} + \frac{2\pi^3 E d^4}{64 L^2}$$

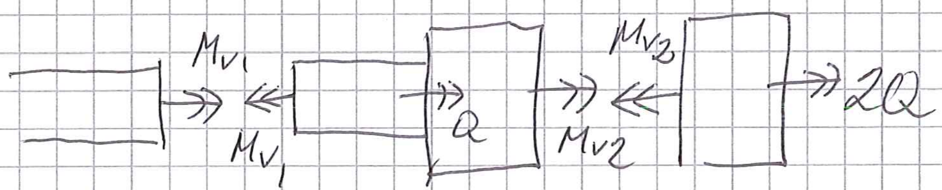
$$= \frac{\pi^3 d^4 E}{L^2}\left(\frac{1}{20 \cdot 16} + \frac{1}{32}\right) \approx 0{,}034\,\frac{\pi^3 d^4 E}{L^2}$$

Hållfasthetslära Z2 TME017 2014-08-21

2)



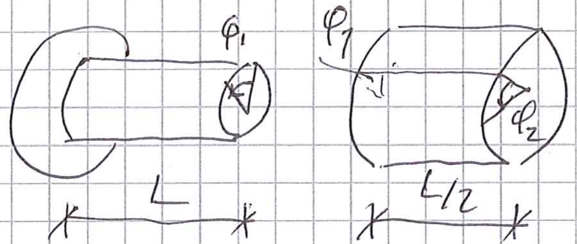
Gör två snitt, inför snittmoment i ① och ②



Momentjämvikt $M_{v1} = M_{v2} + Q$

$$M_{v2} = 2Q \implies M_{v1} = 3Q$$

Deformationssamband



högra ändans rotation

$$\varphi = \varphi_1 + \varphi_2$$

Materialsamband, LB p 57

$$\varphi_1 = \frac{M_{v1} L}{G K_1} \quad \varphi_2 = \frac{M_{v2} L}{2 G K_2} \; , \quad K = \frac{\pi}{2} R^4$$

Med $R_1 = 20$ mm, $R_2 = 30$ mm fås

Hållfasthetslära Z2 TME017 2014-08-21

2) Forts

$$\varphi = \varphi_1 + \varphi_2 = \frac{3QL\cdot 2}{G\pi R^4} + \frac{2Q\cdot L\cdot 2}{2G\pi(1.5R)^4} = \frac{QL}{G\pi R^4}\left(6+\frac{16}{81}\right)$$

$$\varphi = \frac{0.5\cdot 10^3\cdot 1}{7\cdot 10^{10}(2\cdot 10^{-2})^4\cdot\pi}\left(6+\frac{16}{81}\right) = 0.088 \text{ radianer}$$

Maximal skjuvspänning på ytterranden av resp del, se LB p 57

$$\tau_{max} = \frac{M_v}{W_v} = \frac{2M_v}{\pi R^3} \Rightarrow$$

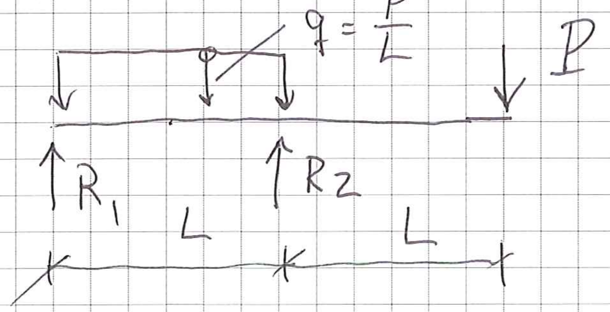
$$\tau_{max_1} = \frac{2\cdot 3Q}{\pi R^3} = \frac{6\cdot 0.5\cdot 10^3}{\pi(2\cdot 10^{-2})^3} = 119 \text{ MPa}$$

$$\tau_{max_2} = \frac{2\cdot 2Q}{\pi(1.5R)^3} = \frac{4\cdot 0.5\cdot 10^3}{\pi(3\cdot 10^{-2})^3} = 24 \text{ MPa}$$

dvs ingen plasticering

Hållfasthetslära Z2 TME017 2014-08-21

3) Bestäm först stödreaktioner



$\uparrow$: $R_1 + R_2 - P - P = 0$ 1)

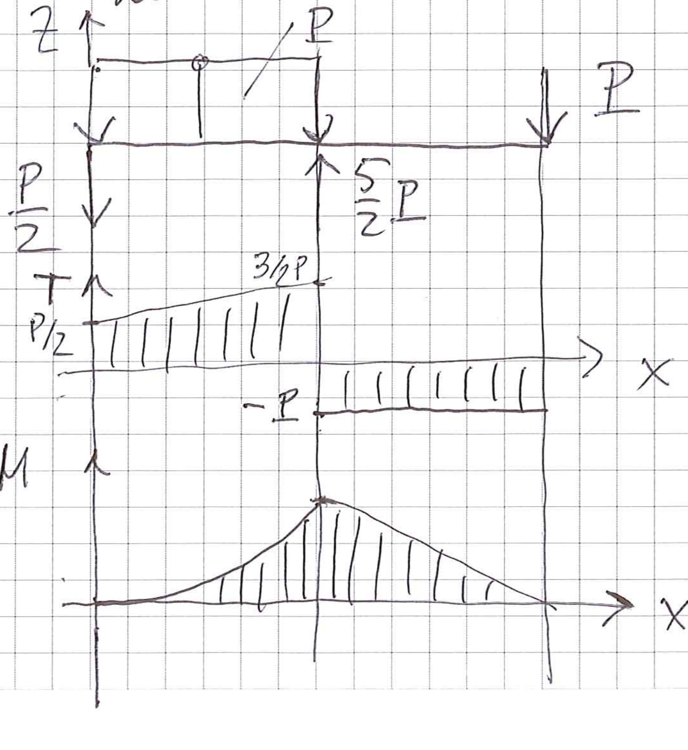
$\curvearrowleft$: $R_2 L - \frac{P}{L} \cdot L \cdot \frac{L}{2} - P 2L = 0$ 2)

$\Rightarrow$ $R_2 = \frac{5}{2} P$, $R_1 = -\frac{1}{2} P$

Använd LB p 70

$$\frac{dT}{dx} = -q(x) \Rightarrow T(x_2) - T(x_1) = -\int_{x_1}^{x_2} q(x)\,dx$$

$$\frac{dM}{dx} = T \Rightarrow M(x_2) - M(x_1) = \int_{x_1}^{x_2} T(x)\,dx$$



$T_{max} = \frac{3}{2} P$

$M(L) = \frac{P}{2} L + \frac{1}{2} P \cdot L = PL$

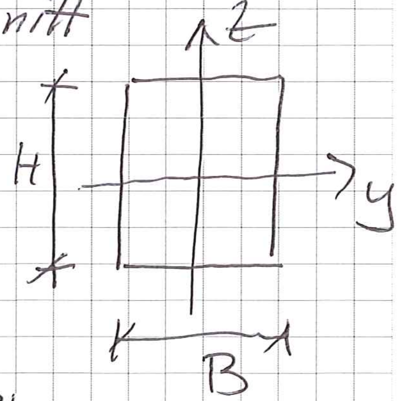
$= M_{max}$

Hållfasthetslära Z2 TME017 2014-08-21

3) forts

max σ: se LB p 78–80

För kvadratiskt tvärsnitt



$$I_y = \frac{BH^3}{12}$$

$$\sigma_{max} = \frac{M_{max} \cdot z_{max}}{I_y} = \frac{PL \cdot H \cdot 12}{2 \cdot B \cdot H^3} = 6\,\frac{PL}{BH^2} \quad \text{för } z = \frac{H}{2}$$

max τ: se LB p 81

$$\tau_{max} = \frac{3}{2}\,\frac{T_{max}}{BH} = \frac{3}{2} \cdot \frac{3}{2}\,\frac{P}{BH} = \frac{9}{4}\,\frac{P}{BH}$$

för $z = 0$

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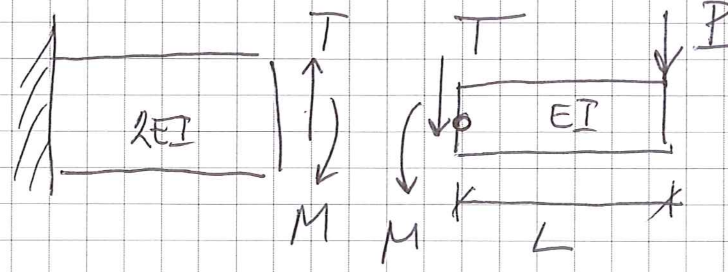
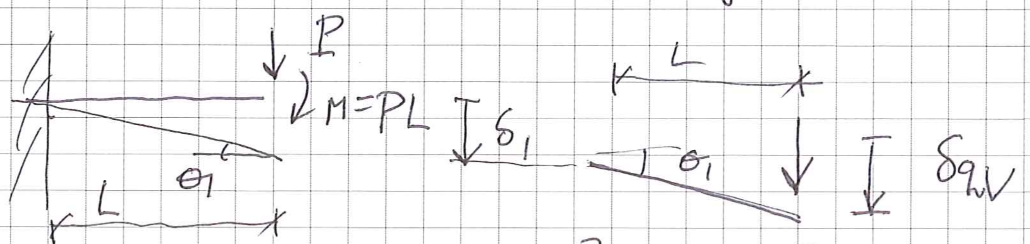
4) Dela upp balken och inför snittkrafter



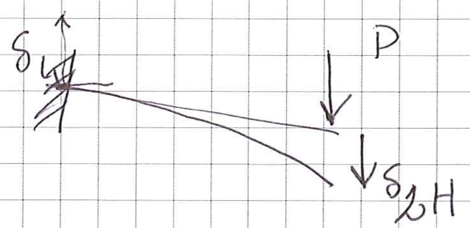
$\uparrow$ $T = -P$

$\curvearrowleft$ $M = PL$

Använd superposition av elementarfall för att bestämma nedböjningen vid P



$$\delta_{2V} = \delta_1 + \theta_1 L = \left(\frac{PL^3}{3\cdot 2EI} + \frac{PL\cdot L^2}{2\cdot 2EI}\right) + L\left(\frac{PL^2}{2\cdot 2EI} + \frac{PL\cdot L}{2EI}\right)$$



$\delta_{2H} \approx$ vertikal

$$\delta_{2H} = \frac{PL^3}{3EI}$$

Total nedböjning

$$\delta_2 = \delta_{2V} + \delta_{2H} = \frac{PL^3}{EI}\left[\left(\frac{1}{6}+\frac{1}{4}\right) + \left(\frac{1}{4}+\frac{1}{2}\right) + \frac{1}{3}\right]$$

$$\delta_2 = \frac{3}{2}\,\frac{PL^3}{EI}$$

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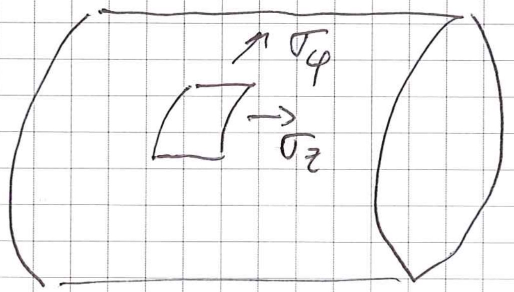
Hållfasthetslära Z2 TME017 2014-08-21

5) Spänningar pga p, LB p181

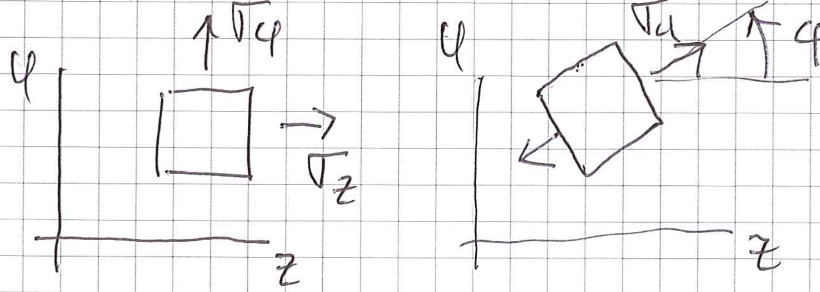


$$\sigma_z = \frac{pa}{2h}$$

$$\sigma_\varphi = \frac{pa}{h}$$

$$\sigma_r \simeq 0 \qquad (\tau_{z\varphi} = 0)$$

Spänning i riktning φ mot z-axeln, LB p171



$$\sigma_\varphi = \frac{\sigma_z + \sigma_\varphi}{2} + \frac{\sigma_z - \sigma_\varphi}{2}\cos 2\varphi + \tau_{z\varphi}\sin 2\varphi$$

$$= \frac{pa}{h}\left(\frac{1 + 1/2}{2} + \frac{1/2 - 1}{2}\cdot\frac{1}{2}\right) = \frac{pa}{h}\cdot\frac{5}{8}$$

$$\sigma_\varphi = 133 \text{ MPa} \Rightarrow \quad p = \frac{\sigma_\varphi \cdot h}{a}\,\frac{8}{5} = \frac{133\cdot 10\cdot 8}{225\cdot 5} = 9.5 \text{ MPa}$$