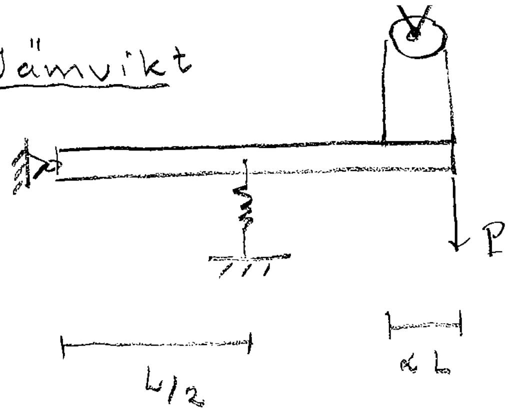
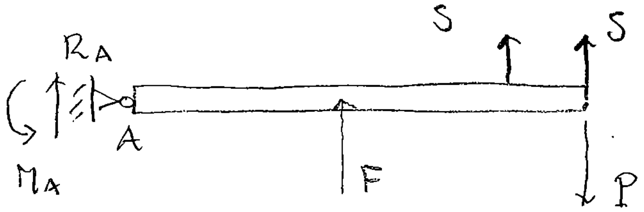
Jämvikt





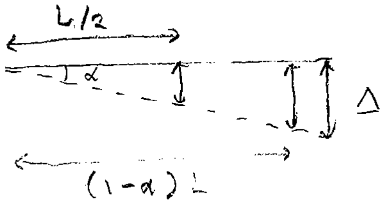
$$\overset{\curvearrowright}{A}: \; -F \cdot L/2 + P \cdot L - S \cdot L - S(1-\alpha)L = 0$$

$$\Rightarrow F = 2P - 2S - 2S(1-\alpha) = 2P - (4-2\alpha)S \qquad (1)$$

Konstitutiva samband

$$F = k \cdot \delta_F \qquad S = \frac{EA}{L_s}\delta_s \qquad (2)$$

Kompatibilitet (antag nedböjning Δ)



$$\begin{cases} \delta_F = \Delta/2 \\ \delta_s = \Delta + \Delta(1-\alpha) = \Delta(2-\alpha) \end{cases}$$

$$\Rightarrow \delta_s = 2\,\delta_F \cdot (2-\alpha) \overset{(2)}{\Rightarrow} \frac{S \cdot L_s}{EA} = 2 \cdot \frac{F}{k}(2-\alpha) \Rightarrow$$

$$S = 2 \cdot \frac{F \cdot EA}{L_s \cdot k}(2-\alpha) \quad \text{insatt i (1)} \Rightarrow$$

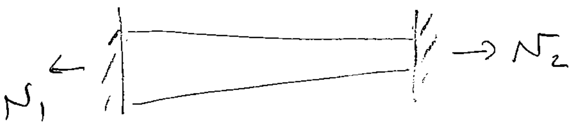
$$F = 2P - (4-2\alpha)\frac{2 \cdot FEA}{L_s \cdot k}(2-\alpha) \Rightarrow$$

$$F\left(1 + 4 \cdot (2-\alpha)^2 \frac{EA}{L_s k}\right) = 2P \Rightarrow F = \frac{2P}{1 + 4(2-\alpha)^2 \frac{EA}{L_s k}} \quad //$$

Konstitutivt samband

$$\varepsilon = \frac{\sigma}{E} + \alpha \Delta T \qquad (1)$$

Jämvikt $\quad dN/dx = 0 \quad \Rightarrow N(x) = \text{konstant} = N$



$\rightarrow: N_1 - N_2 = 0 \Rightarrow N_1 = N_2 = N$

Spänningen

$$\sigma(x) = \frac{N}{A(x)} = \frac{4N}{\pi d(x)^2} \qquad (2)$$

Samband förlängning - töjning:

$$\frac{du}{dx} = \varepsilon(x) \overset{(1)}{=} \frac{\sigma(x)}{E} + \alpha \Delta T \overset{(2)}{=} \frac{N}{E A(x)} + \alpha \Delta T$$

$$\Rightarrow u(x) = \int \frac{N}{EA(x)} dx + \alpha \Delta T x + C =$$

$$= \frac{4N}{\pi E} \int \frac{dx}{d_0^2 (1 - x/2L)^2} + \alpha \Delta T x + C =$$

$$= \frac{4N}{\pi E d_0^2} \left(2L \, (1 - x/2L)^{-1} \right) + \alpha \Delta T x + C$$

Randvillkor:

$$u(0) = 0 \Rightarrow \frac{8NL}{\pi E d_0^2} + C = 0 \Rightarrow C = -\frac{8NL}{\pi E d_0^2}$$

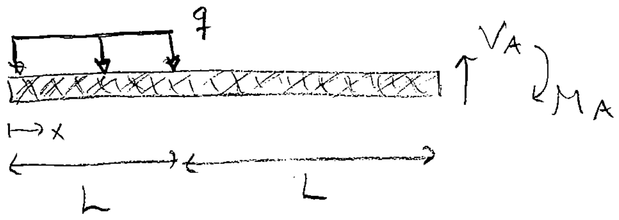
Kompatibilitet:

$$u(L) = 0 \Rightarrow \frac{4N}{\pi E d_0^2} 2L \cdot 2 + \alpha \Delta T \cdot L - \frac{8NL}{\pi E d_0^2} = 0$$

$$\Rightarrow \frac{N\cancel{L}}{\pi E d_0^2} (16 - 8) = -\alpha \Delta T \cancel{L} \Rightarrow N = -\frac{\alpha \Delta T \pi E d_0^2}{8}$$

Spänningen blir: $\quad \sigma(x) = -\frac{1}{2} \frac{\alpha \Delta T E \cancel{d_0^2}}{\cancel{d_0^2} (1 - x/2L)^2} = -\frac{\alpha \Delta T E}{2(1 - x/2L)^2}$

Stödreaktioner

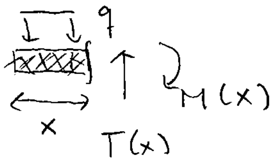


$$\uparrow: \; -q \cdot L + V_A = 0 \Rightarrow \boxed{V_A = q \cdot L}$$

$$\overset{\curvearrowright}{A}: \; M_A - q \cdot L \cdot (L + L/2) = 0 \Rightarrow \boxed{M_A = qL^2 \cdot 3/2}$$

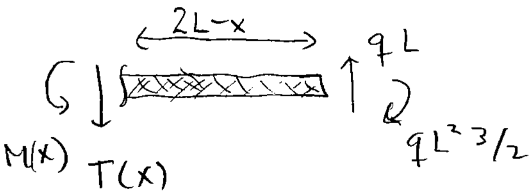
Böjmomentdiagram:

snitta $0 < x < L$



$$\overset{\curvearrowright}{x}: \; M(x) - q \cdot x \cdot x/2 = 0 \Rightarrow M(x) = qx^2/2$$

snitta $L < x < 2L$



$$\overset{\curvearrowright}{x}: \; -M(x) + qL^2 3/2 - qL(2L - x) = 0$$

$$\Rightarrow M(x) = -qL^2/2 + qLx$$

$\therefore$ Max böjmoment fås vid $x = 2L$ (kan inses med det-samma) $M_{b,max} = qL^2 \, 3/2$

Yttröghetsmomentet (enl FS)

$$I_y = \frac{tH^3}{12} + \frac{tBH^2}{2}$$

Max böjnormalspänning:

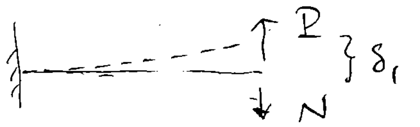
$$\sigma_{b,max} = \frac{M_{b,max} \cdot H/2}{I_y} < 150 \text{ [MPa]} \Rightarrow t > 12{,}9 \text{ mm}$$

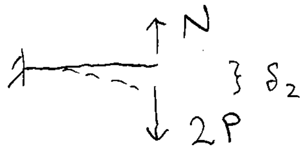
Max utböjning fås m.h.a. superposition av elementarfall 6.4 i FS

$$\delta_1 = \frac{q \cdot (2L)^4}{8\,EI} - \frac{q\,L^3 \cdot (2L)}{24\,EI}\left[4 - \frac{L}{2L}\right] =$$

$$= \frac{qL^4}{EI}\left[2 - \frac{2}{24} \cdot \frac{7}{2}\right] = \frac{qL^4}{EI} \cdot \frac{41}{24} \approx 1{,}6 \text{ m}$$

Låt kraften i linan vara N $\Rightarrow$





Mha elementarfall 6.4 fås:

$$\delta_1 = \frac{(P-N)(2L)^3}{3EI} \qquad \delta_2 = \frac{(2P-N)L^3}{3EI}$$

Kompatibilitet:

linans förlängning $\quad \delta = \delta_1 + \delta_2 = \frac{L^3}{EI}\left(\frac{10}{3}P - 3N\right) \quad (*)$

Konstitutivt samband för linan:

$$\delta = \frac{NL}{EA} \overset{(*)}{=} \frac{L^3}{EI}\left(\frac{10}{3}P - 3N\right) \Rightarrow N = \frac{10P}{3\left(3 + \frac{I}{AL^2}\right)}$$

//

a) Spänningsmatrisen

$$\underline{S} = \begin{bmatrix} 225 & 37{,}5 & 0 \\ 37{,}5 & -75 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{ MPa}$$

Huvudspänningar σ_i fås ur $\det(\underline{S} - \sigma_i \underline{I}) = 0$

$$(225-\sigma_i)(-75-\sigma_i)(-\sigma_i) - 37{,}5^2(-\sigma_i) = 0$$

$$\Rightarrow \begin{cases} \sigma_1 = 0 \\ -18281{,}25 - 150\,\sigma_i + \sigma_i^2 = 0 \end{cases} \Rightarrow$$

$$\sigma_i = 75 \pm \sqrt{75^2 + 18281{,}25} \approx 75 \pm 155 \text{ MPa}$$

$\therefore$ $\sigma_2 \approx 230$ MPa $\sigma_3 \approx -80$ MPa

(alt. använd ekv. i 7. FS)

b) En huvudriktning sammanfaller med z-axeln (ty $\tau_{xz} = \tau_{yz} = 0$) dvs $\underline{n}_1 = [0\ 0\ 1]^T$

Vi kan använda 7 i FS (plant spänningstillstånd)

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \approx 155 \text{ MPa}$$

$$\sin(2\varphi) = \frac{\tau_{xy}}{R} \approx 0{,}243 \Rightarrow \varphi = 7^\circ \text{ eller } 97^\circ$$

dvs

$$\underline{n}_1 = [0\ 0\ 1]^T$$

$$\underline{n}_2 = [\cos(7^\circ)\ \sin(7^\circ)\ 0]^T \approx [0{,}99\ 0{,}12\ 0]^T$$

$$\underline{n}_3 = [\cos(97^\circ)\ \sin(97^\circ)\ 0]^T \approx [-0{,}12\ 0{,}99\ 0]^T$$

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c) von Mises enl FS 9:

$$\sigma_e = \frac{1}{\sqrt{2}}\sqrt{(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2} \approx 278 \text{ MPa}$$

Tresca

$\sigma_e \approx 309$ MPa   dvs det flyter enl Tresca men inte Mises