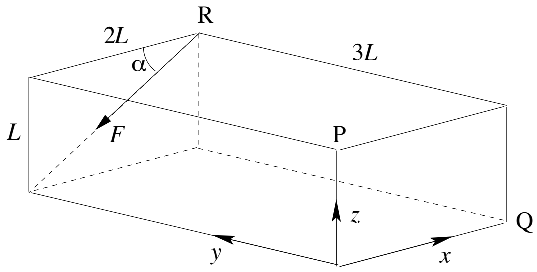
Lösningsförslag dugga 081118

1.



a)

$$\boldsymbol{F} = (-F\cos\alpha,\ 0,\ -F\sin\alpha) = \frac{F}{\sqrt{5}}(-2,\ 0,\ -1)$$

b)

$$\boldsymbol{M}_{\mathrm{P}} = \overrightarrow{\mathrm{PR}} \times \boldsymbol{F} = \begin{vmatrix} \boldsymbol{e}_x & \boldsymbol{e}_y & \boldsymbol{e}_z \\ 2L & 3L & 0 \\ -2F/\sqrt{5} & 0 & -F/\sqrt{5} \end{vmatrix} = \frac{FL}{\sqrt{5}}(-3,\ 2,\ 6).$$

c)

$$M_{\mathrm{PQ}} = \boldsymbol{M}_{\mathrm{P}} \cdot \boldsymbol{e}_{\mathrm{PQ}},$$

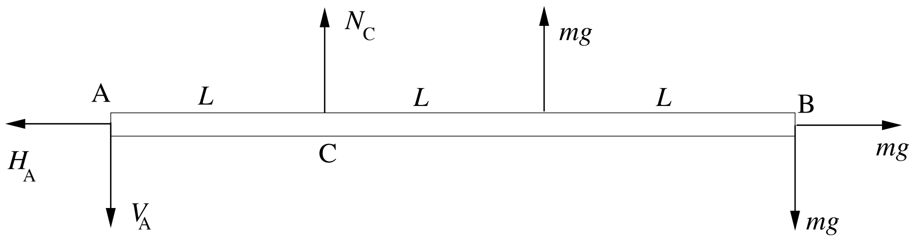
där $\boldsymbol{e}_{\mathrm{PQ}}$ är en enhetsvektor längs PQ. Denna bestäms ur sambandet

$$\boldsymbol{e}_{\mathrm{PQ}} = \frac{\overrightarrow{\mathrm{PQ}}}{|\overrightarrow{\mathrm{PQ}}|} = \frac{(2L,\ 0,\ -L)}{\sqrt{(2L)^2 + L^2}} = \frac{1}{\sqrt{5}}(2,\ 0,\ -1).$$

Insättning ger det sökta momentet

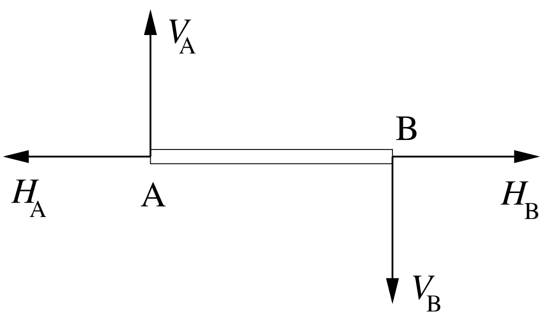
$$M_{\mathrm{PQ}} = \frac{FL}{5}(-3,\ 2,\ 6) \cdot (2,\ 0,\ -1) = -\frac{12}{5}FL.$$

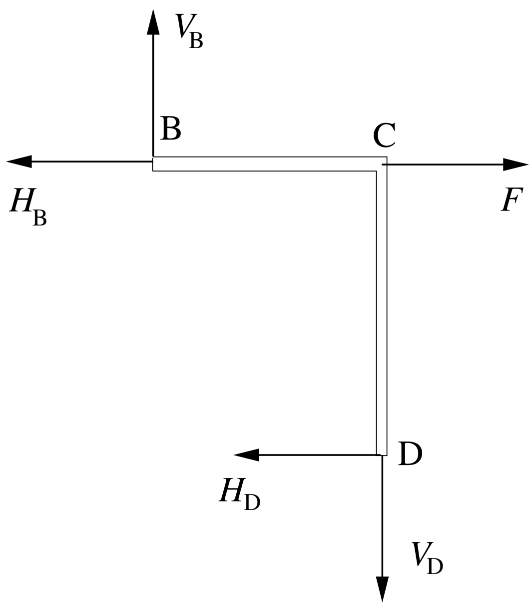
2.



$$\leftarrow \qquad H_{\rm A} - mg = 0,$$
$$\downarrow \qquad V_{\rm A} - N_{\rm C} - mg + mg = 0,$$
$$\overset{\curvearrowright}{\rm A} \qquad N_{\rm C}L + mg \cdot 2L - mg \cdot 3L = 0.$$

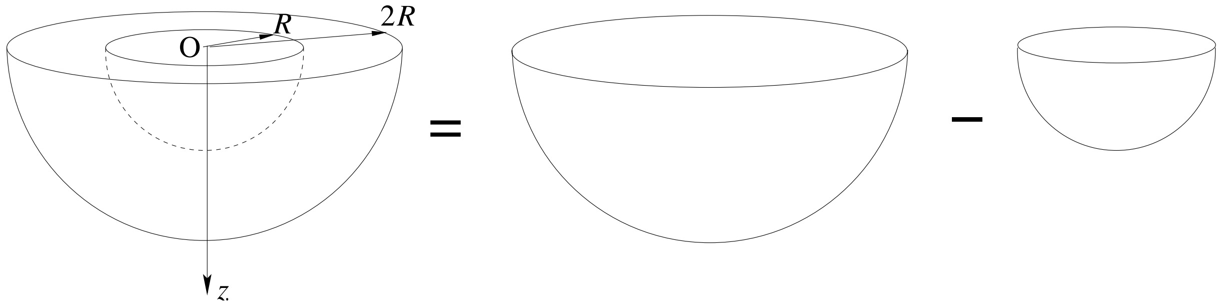
3.





4.

Skivan kan ses som skillnaden mellan en halvsfär med radien $2R$ (element nr 1) och en med radien R (element nr 2).



Element nr	Volym V_i	$\overline{z}_i$	$V_i\overline{z}_i$
1	$\frac{2}{3}\pi(2R)^3$	$\frac{3}{8}\cdot 2R$	$4\pi R^4$
2	$\frac{2}{3}\pi R^3$	$\frac{3}{8}R$	$\frac{1}{4}R^4$
$1-2$	$\frac{14}{3}\pi R^3$	?	$\frac{15}{4}\pi R^4$

Skålens tyngdpunkts z-koordinat fås sedan som

$$\overline{z} = \frac{\frac{15}{4}\pi R^4}{\frac{14}{3}\pi R^3} = \frac{45}{56}R.$$

5.

a)

$$a = \frac{\mathrm{d}v}{\mathrm{d}t} = v_0\omega \sin\omega t.$$

b)

$$\frac{\mathrm{d}s}{\mathrm{d}t} = v = v_0 \sin\omega t \quad \Rightarrow \quad \int_{s_0}^{s} \mathrm{d}s = \int_0^t v_0 \sin\omega t \mathrm{d}t \quad \Rightarrow \quad s = s_0 + \frac{v_0}{\omega} - \frac{v_0}{\omega}\cos\omega t.$$