(1/2)

Exam solution manual

(12/01/15 SSY285)

Q1) $G(s) = C(sI-A)^{-1}B + D = \begin{bmatrix} 1 & 0 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} s+1 & -1 \\ -3 & s+1 \end{bmatrix}^{-1} \begin{bmatrix} -1 & 1 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

$G(s) = \dfrac{1}{s^2+2s-2} \begin{bmatrix} -s-1 & s(s+3) \\ s-2 & 2 \end{bmatrix}$ (1p) $\Rightarrow \det G(s) = \dfrac{-s-1}{s^2+2s-2}$

$z = 1$; $p_1 = 0{,}73$, $p_2 = -2{,}73$

not a minimum phase, not stable (1p)

Q2)

a) $A = \begin{bmatrix} 0 & a_2 + k_1 a_3 \\ a_1 & k_1 a_4 \end{bmatrix}$; $B = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$ $C = \begin{bmatrix} C & 0 \end{bmatrix}$; $D = C$ (1p)

b) $\det(\lambda I - A) = \begin{vmatrix} \lambda & -1-k_1 \\ -1 & \lambda - 2k_1 \end{vmatrix} = \lambda(\lambda - 2k_1) - (1+k_1) =$

$\lambda^2 - 2k_1\lambda - (1+k_1) =$

$s^2 + 2s + 0$

$k_1 = -1$ (1p)

c) $R_2 = \begin{bmatrix} B & AB \end{bmatrix}\Big|_{k_1=-1} = \begin{bmatrix} b_1 & -2(1+k_1) \\ -2 & b_1 - 4k_1 \end{bmatrix}$

$\det(R)\Big|_{k_1=-1} = b_1^2 + 4b_1 = 0 \Rightarrow$

We loos contr. if $b_1 = \begin{cases} 0 \\ -4 \end{cases}$

$\mathcal{O}_2 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \Rightarrow$ rank deficiency $\Rightarrow$ not observable

Not a minimal order realization and independent of b_1 ! (1p)

d) $A = \begin{bmatrix} 0 & 0 \\ 1 & -2 \end{bmatrix}$; $B = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$; $C = \begin{bmatrix} 1 & 0 \end{bmatrix}$; $D = 1$;

$\operatorname{eig}(A) \Rightarrow \begin{bmatrix} -2 \\ 0 \end{bmatrix} \Rightarrow$ e.g. $V_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$; $V_2 = \begin{bmatrix} 0{,}89 \\ 0{,}44 \end{bmatrix}$ eigenvectors are not unique $\tilde{B}, \tilde{C}$!

$T^{-1} = \begin{bmatrix} 0 & 0{,}89 \\ 1 & 0{,}44 \end{bmatrix}$ (1p) $\Rightarrow T = \dfrac{\operatorname{adj}(T^{-1})}{\det(T^{-1})} = \begin{bmatrix} -0{,}5 & 1 \\ 1{,}12 & 0 \end{bmatrix}$ (1p)

$\tilde{A} = TAT^{-1} = \begin{bmatrix} -2 & 0 \\ 0 & 0 \end{bmatrix}$; $\tilde{B} = TB = \begin{bmatrix} -2{,}5 \\ 1{,}18 \end{bmatrix}$; $\tilde{C} = \begin{bmatrix} 0, & 0{,}89 \end{bmatrix}$, $D = \tilde{D}$ (1p)

e) $y(t) = \tilde{C} e^{\tilde{A}t} x_0 = \begin{bmatrix} 0 & 0{,}89 \end{bmatrix} \begin{bmatrix} e^{-2t} & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \Rightarrow y(10) = y(20) = 0{,}89$ (1p)

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$$\bar{P} = \begin{bmatrix} p_1 & p_2 \\ p_2 & p_3 \end{bmatrix}$$

Q3) $\bar{P} - A\bar{P}A^T - NVN^T + \underbrace{A\bar{P}C^T(W + C\bar{P}C^T)^{-1}C\bar{P}A^T}_{X} = 0$

a) $\begin{bmatrix} -2p_2 - p_3 - 1 + p_2(p_1 + p_2)/p_1 + p_1 & -p_3 - 1 + X \\ -p_3 + p_2^2/p_1 + p_2 + 1 & -1 + p_2^2/p_1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$\Rightarrow p_1 = p_2 = p_3 = 1 \qquad \bar{P} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \bar{P}^T \geq 0$

$\bar{L} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

b) $A - \bar{L}C = \begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix} \Rightarrow (\lambda I - A + \bar{L}C) \rightarrow \lambda_1 = \lambda_2 = 0$

Q4) $A = 1$; $B = [1 \ 1]$, $Q_x = 1$, $Q_u = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

discrete-time LQR; discrete-time CARE

$A^TPA - P - A^TPB(Q_u + B^TPB)^{-1}B^TPA + Q_x = 0$

$\Rightarrow P - P - P[1 \ 1]\left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} P [1 \ 1]\right)^{-1}\begin{bmatrix} 1 \\ 1 \end{bmatrix} P + 1 = 0$

a) $2p^2 - 2p - 1 = 0 \qquad p = 1,36 \qquad K = (B^TPB + Q_u)^{-1}B^TPA = \begin{bmatrix} 0,36 \\ 0,36 \end{bmatrix}$ (1p)

b) $A - BK = 1 - 2 \cdot 0,36 = 0,267$ (1p)

Q5) $G_{yr} = \dfrac{G_n F}{1 + G_n C} = \dfrac{3}{(s+1)(s+2+3C(s+3))}$ (1p)

$y_\infty = \lim\limits_{s \to 0} G_{yr} \cdot \dfrac{1}{s} = \dfrac{3}{2 + 9C} \Rightarrow C = \dfrac{1}{9}$

$= G_n + \Delta_a \rightarrow \Delta_a = \dfrac{1 - 3(s+3)(s+2)}{(s+1)(s+2)^2}$ (1p)

$= (1 + \Delta_m)G_n \rightarrow \Delta_m = \dfrac{\Delta_a}{G_n} = \dfrac{1 - 3(s+3)(s+2)}{3(s+3)(s+2)}$ (1p)

Q6) $b = \sqrt{2}$, $b_1 = 0,5$, $r = 0,5$, $a = -1,5 \qquad q_x = ?$

$u = -5\sqrt{2}x = -Q_u^{-1} \cdot B^T \cdot \bar{P}x \Rightarrow \bar{P} = 5$ (1p) $\qquad q_u = 1$ (1p)