Solution to exam

(12/17 2012)

Q1)

a) $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 0 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} -1 & 0{,}5 \\ -0{,}5 & 0{,}5 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$; $\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$

b) $G(s) = C(sI - A)^{-1} B + D = \begin{bmatrix} \frac{s}{s+2} & \frac{1}{s+2} \\ \frac{1}{s+3} & \frac{1}{s+3} \end{bmatrix}$

c) $z = 1$

$p_1 = -2,\ p_2 = -3$

d) non-minimum because of $Re(z) > 0$

stable because of $\forall i \ Re(p_i) < 0$

Q2)

a) $R_2 = \begin{bmatrix} 0 & -1 \\ 2 & 0 \end{bmatrix}$; $\det(R_2) = 2 \neq 0$ for any possible α

this is α independent!

b) $O_2 = \begin{bmatrix} 2 & \frac{1}{\alpha} \\ -4 & -1 \end{bmatrix}$; $\det O_2 = -2 + \frac{4}{\alpha} = 0 \Rightarrow$ if $\alpha = 2$ then the system is not observable

c) $\det(\lambda I - A) = \det \begin{bmatrix} \lambda + 1{,}5 & 0{,}5 \\ \alpha & \lambda \end{bmatrix} = (\lambda + 1{,}5)\lambda - 0{,}5\alpha = 0$

designed pole conf: $(\lambda + 0{,}5)(\lambda + 1) = \lambda^2 + 1{,}5\lambda + \underbrace{0{,}5}_{-0{,}5\cdot\alpha \,=\, 0{,}5} = 0$

$-0{,}5 \cdot \alpha = 0{,}5 \Rightarrow$

$\alpha = -1$

d) $A(\alpha = -1) = \begin{bmatrix} -1{,}5 & -0{,}5 \\ 1 & 0 \end{bmatrix}$

$T = \tilde{R}_2 R_2^{-1} = \begin{bmatrix} 1 & -1{,}5 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 2 & 0 \end{bmatrix}^{-1} = \begin{bmatrix} 1{,}5 & 0{,}5 \\ -1 & 0 \end{bmatrix}$

$\hat{A} = TAT^{-1} = \begin{bmatrix} -1{,}5 & -0{,}5 \\ 1 & 0 \end{bmatrix}$

$\hat{B} = TB = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$\hat{D} = D$, $\hat{C} = C \cdot T^{-1} = [-2 \quad -5]$

Q3)

a) $A = 1$; $B = [1 \quad 1]$

$Q_x = \frac{1}{10}$; $Q_u = \begin{bmatrix} 100 & 0 \\ 0 & 100 \end{bmatrix}$

CARE: $A^T\bar{P} + \bar{P}A + Q_x - \bar{P}BQ_u^{-1}B^T\bar{P} = 0$

scalar riccati equation is obtained;

$1 \cdot \bar{p} + \bar{p} \cdot 1 + \frac{1}{10} - \bar{p}^2 \cdot \underbrace{[1 \quad 1] \begin{bmatrix} \frac{1}{100} & 0 \\ 0 & \frac{1}{100} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}}_{\frac{2}{100}} =$

$\frac{2}{100} \cdot \bar{p}^2 - 2\bar{p} - \frac{1}{10} = 0$

$\bar{p}_* = 100{,}05 > 0$

$\bar{K} = Q_u^{-1} B^T \cdot \bar{P} = \begin{bmatrix} 1{,}0005 \\ 1{,}0005 \end{bmatrix}$

b)
$$A - B\bar{K} = 1 - \begin{bmatrix}1 & 1\end{bmatrix}\begin{bmatrix}1{,}0005\\ 1{,}0005\end{bmatrix} = -1{,}0010$$

- High weights Q_u reflects high penalty on the control input signal & $\Rightarrow$ expensive LQR strategy

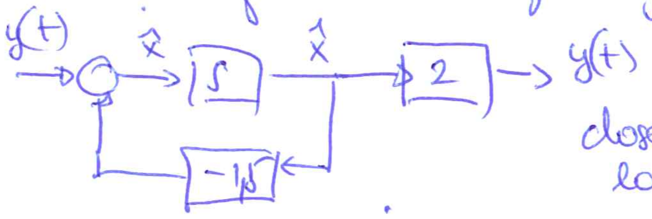
(- Alternatively this can also be seen on the placement of the optimal pole from $+1 \to -1$, LQR mirrors the open-loop poles onto the imaginary axes.)

Q4) Filter ARE: $A\bar{P} + \bar{P}A^T - \bar{P}C^T R_w^{-1} C \bar{P} + W R_v W^T = 0$

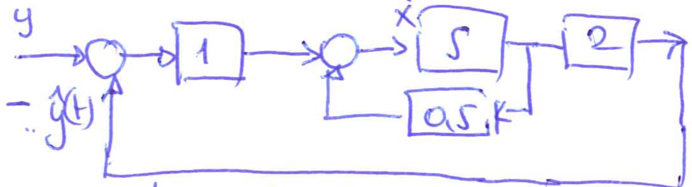
a)
$$0{,}5\bar{p} + 0{,}5\bar{p} - \bar{p}^2 \cdot \cancel{2} \cdot \frac{1}{4} \cdot \cancel{2} + 2 \cdot 0{,}5 \cdot 2 = 0$$
$$\bar{p}^2 - \bar{p} - 2 = (\bar{p} - 2)(\bar{p} + 1) \quad ✓$$
$$\rightarrow \bar{p} = 2 > 0$$

$$\bar{L} = \bar{P} \cdot C^T R_w^{-1} = 2 \cdot 2 \cdot \frac{1}{4} = 1$$

b) $A - \bar{L}C = 0{,}5 - 1 \cdot 2 = -1{,}5$ // Open or closed loop block diagrams are equally ok



closed loop



open

Q5)

a)
$$G_{yr}(s) = \frac{F(s) \cdot C(s)(1+\Delta_m)G_n(s)}{1 + C(s)(1+\Delta_m)G_n(s)} = \frac{\frac{1}{s+c} \cdot 3(s+2)}{1 + 3(s+2)} = \frac{3(s+2)}{(s+c)(3s+7)}$$

b)
$$y_\infty = r_\infty = 1 = \lim_{s\to 0} s \cdot Y(s) = \lim_{s \to 0} s \cdot G_{yr}(s) \cdot \frac{1}{s} = \frac{6}{7c} = 1; \quad c = \frac{6}{7}$$

d)
$$G = \begin{bmatrix} G_{yr} & G_{yd} \\ G_{zr} & G_{zd} \end{bmatrix} = \begin{bmatrix} \frac{3(s+2)}{(s+\frac{6}{7})(3s+7)} & \frac{1}{3s+7} \\ \frac{s+1}{(s+\frac{6}{7})(3s+7)} & \frac{s+1}{(3s+7)} \end{bmatrix} = \begin{bmatrix} G_{yr} & \frac{1}{1+G_n(\Delta_m+1)C} \\ C \cdot F \cdot S(s) & -CS \end{bmatrix}$$

$$\frac{1}{1 + G_n(\Delta_m + 1) \cdot C} = S(s)$$

e)
$$G_n(1+\Delta_m) = G_n + \underbrace{\Delta_a} = G_n + \underbrace{G_n \cdot \Delta_m}$$
$$\Delta_a = G_n \cdot \Delta_m = \frac{3(s+2) \cdot s}{(s+1)^2}$$