# Discrete Event Control and Optimization 

## Exam SSY 220, Tuesday May 28, 08:30-12:30, M <br> Teacher: Martin Fabian, (772) 3716

Time when teacher present: 09:30, 11:30


Solutions and answers should be complete, written in English and be unambiguous and well motivated. In the case of ambiguously formulated exam tasks, the suggested solution with possible assumptions must be motivated. The examiner retains the right to accept or decline the rationality of assumptions and motivations.
In total the exam comprises 25 credits. For the grades 3,4 and 5 , is respectively required 10 , 15 and 20 credits.
Solutions will be announced on the course web-page on the first week-day after the exam date. Exam results are announced through Chalmers’ administrative routines. The corrected exams are open for review seven work days after the exam, 12:30-13:30 at the department.

## Task 1. Controllability

Controllability is an important property in the Supervisory Control Theory; the supervisor must be controllable with respect to the plant and the uncontrollable events. Below is a specification $K$ and a plant component $G_{i}$ of which there are two ( $i=1,2$ ), and the uncontrollable events are !b1, and !c2 (note that the plant components are not identical in this respect). By a clever trick called plantify we can remove the distinction between plant and specification while still being able to synthesize a proper supervisor.

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a) Explain controllability and give the formal definition. What can happen if controllability does not hold?
b) Plantify the given specification $K$ above.
c) Is $K$ controllable with respect to the $G_{i}(i=1,2)$ and the uncontrollable events !b1, and !c2? Explain.

## Task 2. Abstractions for Compositional Verification

Emilia was given the task to calculate a supervisor by compositional synthesis. One of the automata looked like the one to the left below. Emilia happily abstracted away the local uncontrollable event (!u2) into the automaton on the right.

a) Is the abstraction valid for controllability verification? Explain.
b) Is the abstraction valid for non-blocking verification? Explain.

## Task 3. Linear Programming

Emil has an integer programming minimization problem that he tries to solve with LPrelaxations and branch and bound. The search tree Emil has generated so far is shown below. The $z_{i}$ (with $i=0-12$ ) values are the current LP-relaxation solutions at the respective node; the values for $z_{5}$ and $z_{9}$ are not disclosed. The nodes are solved in the order of their numbering. The term integer denotes that all variables are integer in the LP-relaxed solution at the respective node. The nodes marked with a slash ( $($ ) are cut off and will not be searched further by Emil.

a) Why are nodes 4,7 , and 11 cut off?
b) Why is node 6 cut off?
c) Why is node 10 cut off?
d) Between which bounds lies $z_{5}$ ?
e) Between which bounds lies $z_{9}$ ?
f) Between which bounds lies the global solution $z^{*}$ ?

## Task 4. Discrete Optimization

Emilia works as a project manager. She has a new project to manage, that she has broken down into sub-tasks with some requirements and estimated times. This is summarized in the table below.

| Sub-task | Time | Requirements |
| :--- | :--- | :--- |
| A | 6 | Not simultaneously with B |
| B | 5 | Not simultaneously with A |
| C | 3 | Can start only after A is done |
| D | 4 | Can start only after A and B are done |
| E | 3 | Can start only after C and D are done |

a) Draw a (precedence) graph that models this project.
b) Using Dijkstra's algorithm, find the least cost path through the graph. Show on each iteration which node is taken out from and put in to the queue, and also what the queue looks like.
c) There are more than one equally good least cost paths for this example. Does Dijkstra's algorithm have any preference for one of them? If so, or not so, explain.


