# *Discrete Event Systems*

## *Course code: SSY165*

# *Examination 2015-10-24*

Time: 8:30-12:30, Location: M-building

Teacher: Bengt Lennartson, phone 3722

The examination includes 25 points, where grade three requires 10 points, grade four 15 points and grade five 20 points.

The result of this examination is announced and inspection of the grading is done on Monday *November 9* and Tuesday *November 10*, 12:30-13:00 at the division.

*Allowed aids at the examination:*

- Standard mathematical tables such as Beta, see also formulas in the end of this examination.
- Pocket calculator.

Good luck!

Department of Signals and Systems
Division of Automatic Control, Automation and Mechatronics
Chalmers University of Technology

**1**

Show the following set implication by relations based on predicate expressions

$$A \subseteq B \Rightarrow (A \setminus D) \cap C \subseteq B \cap C \cap \sim D$$

(4 p)

**2**

Consider a Petri net with three places and two transitions, where the incidence matrix $A^+$, defining the weights of the arcs between the input transitions and their output places, is

$$A^+ = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix},$$

the incidence matrix $A^-$, defining the weights of the arcs between the input places and their output transitions, is

$$A^- = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix},$$

and the initial marking vector

$$m_0 = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^T.$$

a) Generate a Petri net based on the given matrix information.

(1 p)

b) Show that the number of tokens in the second and third places can increase unboundedly by evaluating the corresponding reachability graph.

(2 p)

c) Add an additional so called control place in the Petri net to avoid that the number of tokens in the second place can be more than one. Verify this result by evaluating the reachability graph for the modified Petri net.

(2 p)

**3**

Consider a plant $P$ with the language

$$L(P) = \overline{ac(b+d) + bc(a+ecd) + cd},$$

and a specification $Sp$ given by the marked language

$$L_m(Sp) = ab + ba + d.$$

Assume that the events $d$ and $e$ are uncontrollable, while $a, b$ and $c$ are controllable.

a) Formulate automata for the languages $L(P)$ and $L_m(Sp)$ with minimal number of states.

(1 p)

b) Generate a controllable and nonblocking supervisor, by the fix point algorithm presented in the lecture notes. Show the resulting automaton after each Backward_Reachability (Coreachability) computation.

(3 p)

**4**

To be able to show that the closed loop system $P||S$ is controllable if and only of the supervisor $S$ is controllable, the following result is useful

$$L_A\Sigma \cap L_B\Sigma = (L_A \cap L_B)\Sigma,$$

where $L_A$ and $L_B$ are languages, and $\Sigma$ is a set of events (strings of length one).

a) Show that the language intersection/concatenation expressions above are equal. According to Beta (Math Handbook) $L_1L_2 = \{s_1s_2 | s_1 \in L_1 \wedge s_2 \in L_2\}$.

(2 p)

b) Assume that the alphabets $\Sigma_P = \Sigma_S$, and show that the closed loop system $P||S$ is controllable with respect to the plant, if and only of the supervisor $S$ is controllable also with respect to the plant, i.e.

$$L(P||S)\Sigma_u \cap L(P) \subseteq L(P||S) \;\Leftrightarrow\; L(S)\Sigma_u \cap L(P) \subseteq L(S).$$

The following additional results are then required:

1. $L(P||S) = L(P) \cap L(S)$ when $\Sigma_P = \Sigma_S$,
2. $A \cap B \subseteq C \cap B \;\Leftrightarrow\; A \cap B \subseteq C$,
3. $(L(P)\Sigma_u \cap L(P)) \cap (L(S)\Sigma_u \cap L(P)) \subseteq L(S), \;\Leftrightarrow\; L(S)\Sigma_u \cap L(P) \subseteq L(S)$.

(2 p)

**5**

Consider two one-degree-of-freedom robots with identical continuous dynamics

$$\begin{bmatrix} \dot{\theta}^k \\ \dot{\omega}^k \end{bmatrix} = \begin{bmatrix} \omega^k \\ -d\cos(\theta^k) + u^k \end{bmatrix},$$

where $k = 1, 2$ for the two robots, $d$ is the distance to the center of gravity, and $u^k(t)$ is a continuous control signal for each robot. Also, suppose that these robots are to move through a shared zone. The robots are in this shared zone when $|\theta^k| \leq \pi/4$ and outside otherwise.

The first robot starts out pointing upwards, $\theta^1 = \pi/2$, moves through the zone and ends pointing downwards, $\theta^1 = -\pi/2$, the second robot does the converse. Obviously each robot will have to pass the shared zone.

Formulate a hybrid automaton for the two robots, where both robots being in the shared zone is considered as a forbidden location.

(3 p)

**6**

When a machine is broken there are two alternatives, either the machine is repaired or it is replaced by a new machine. After a machine has been repaired it is a higher risk that the machine is broken in the next time instant. Assume that the conditional probability to be broken for a new machine is $1\%$, while for a repaired machine the risk is $2\%$.

a) Formulate a Markov Decision Process for this problem, where action $a_1$ means that the broken machine is replaced by a new machine, while action $a_2$ means that the machine is repaired. Write a state transition diagram including the two actions.

(2 p)

b) Assume that the cost to buy a new machine is $\alpha$ times higher than to repair the machine. Decide for which values of the relative cost $\alpha$, it is more profitable (lower cost) to repair the machine instead of buying a new one.

(3 p)

Solution DES exam 2015-10-24

BL 151102

1.

$$A \subseteq B \Rightarrow (A \setminus D) \cap C \subseteq B \cap C \cap \sim D \quad ?$$

$$p_A(x) \wedge \neg p_D(x) \wedge p_C(x) \rightarrow p_B(x) \wedge p_C(x) \wedge \neg p_D(x) \Leftrightarrow$$

$$\neg(p_A(x) \wedge \neg p_D(x) \wedge p_C(x)) \vee (p_B(x) \wedge p_C(x) \wedge \neg p_D(x)) \Leftrightarrow$$

$$\neg p_A(x) \vee \neg \underbrace{(p_C(x) \wedge \neg p_D(x))}_{p_*(x)} \vee (p_B(x) \wedge \underbrace{p_C(x) \wedge \neg p_D(x)}_{p_*(x)}) \Leftrightarrow$$

$$\neg p_A(x) \vee (p_B(x) \wedge p_*(x)) \vee \neg p_*(x) \Leftrightarrow$$

$$(\neg p_A(x) \vee p_B(x)) \wedge (\neg p_A(x) \vee p_*(x)) \vee \neg p_*(x) \Leftrightarrow$$

$$\underbrace{(p_A(x) \rightarrow p_B(x))}_{T \text{ since } A \subseteq B} \wedge (\neg p_A(x) \vee p_*(x)) \vee \neg p_*(x) \Leftrightarrow$$

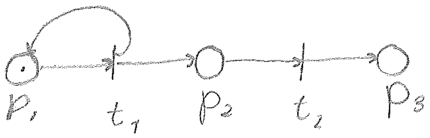
$$\neg p_A(x) \vee \underbrace{p_*(x) \vee \neg p_*(x)}_{T}$$

$$\Leftrightarrow \neg p_*(x) \vee T \Leftrightarrow T$$

$\therefore (A \setminus D) \cap C \subseteq B \cap C \cap \sim D$ when $A \subseteq B$

2 a) $A^+ = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$ $A^- = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$ $m_0 = [1\ 0\ 0]^T$

$m = [m_1\ m_2\ m_2]^T$

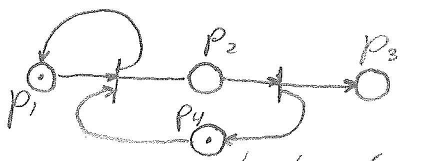



b)

$(100)^T$
$\downarrow$
$(110)^T \rightarrow (101)^T$
$\downarrow \qquad\qquad \downarrow$
$(120)^T \rightarrow (111)^T \rightarrow (102)^T$
$\downarrow \qquad\qquad \downarrow \qquad\qquad \downarrow$
$(130)^T \rightarrow (121)^T \rightarrow (112)^T \rightarrow (103)^T$
$\vdots \qquad \vdots \qquad \vdots \qquad \vdots$

# of tokens in $p_2$ increases without bound since $t_1$ can always be fired.
# of tokens in $p_3$ increases without bound as long as there are tokens in $p_2$.

c)

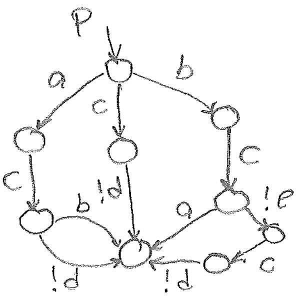



control place
$\Rightarrow m_2 \leq 1$

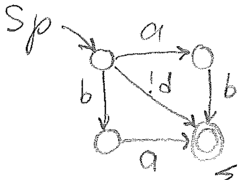
$(1001)^T$
$\downarrow$
$(1100)^T \rightarrow (1011)^T$
$\downarrow$
$(1110)^T \rightarrow (1021)^T$
$\downarrow$
$(1120)^T \rightarrow (1031)^T$
$\vdots$

$\therefore m_2 \leq 1$

3. a)



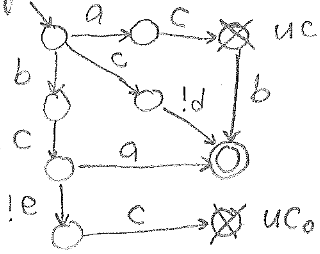

$\Sigma_P = \{a, b, c, d, e\}$



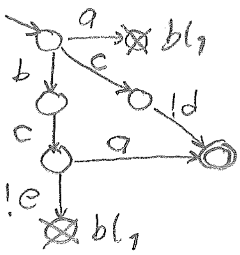

$\Sigma_{Sp} = \{a, b, d\}$

uc = uncontrollable state

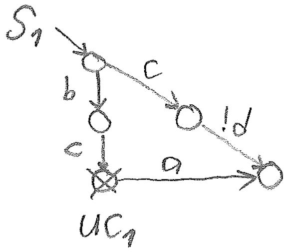
bl = blocking state



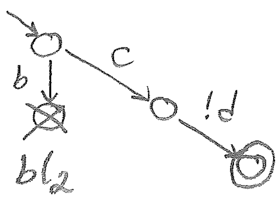

k=1 blocking



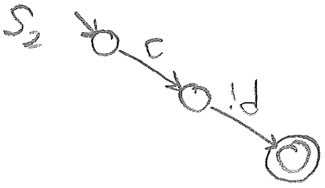

k=1 uncontrollable




k=2 blocking




k=2 uncontrollable




$S = S_3 = S_2$ = maximal permissive controllable and nonblocking supervisor

4.

a) $L_A\Sigma \cap L_B\Sigma = \{se \mid s\in L_A \wedge e\in\Sigma\} \cap$

$\cap \{s'e \mid s'\in L_B \wedge e\in\Sigma\} = \{se \mid s\in L_A \wedge s\in L_B \wedge e\in\Sigma\}$

$= (L_A \cap L_B)\Sigma$

b) $L(P\|S)\Sigma_u \cap L(P) \subseteq L(P\|S) \Leftrightarrow$

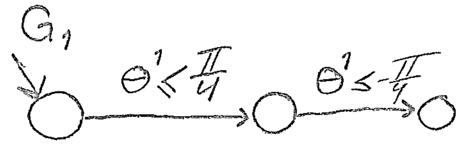
$(L(P)\cap L(S))\Sigma_u \cap L(P) \subseteq L(P)\cap L(S) \Leftrightarrow$

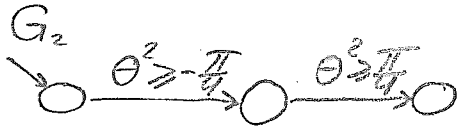
$L(P)\Sigma_u \cap L(S)\Sigma_u \cap L(P) \cap L(P) \subseteq L(S) \Leftrightarrow$

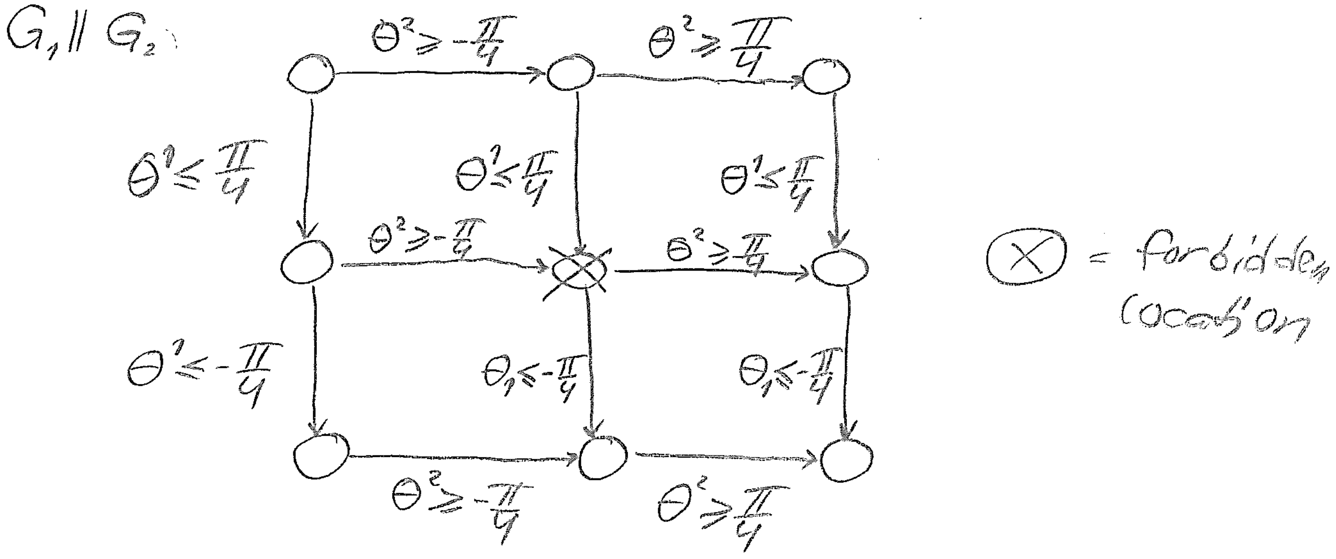
$(L(P)\Sigma_u \cap L(P)) \cap (L(S)\Sigma_u \cap L(P)) \subseteq L(S) \Leftrightarrow$

$L(S)\Sigma_u \cap L(P) \subseteq L(S)$

5.



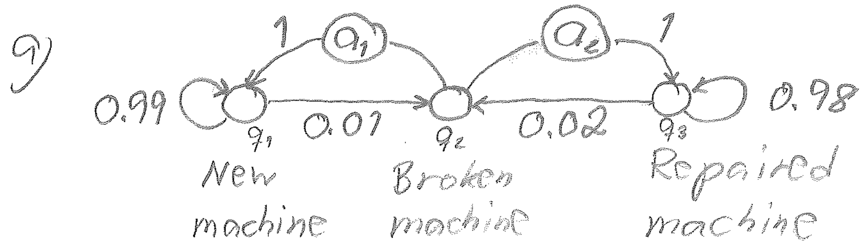




In each location the following differential eq. are valid

$\ddot{\theta}^k = -d\cos\theta^k + u^k \qquad k=1,2$

6.

a)




$p = p\mathbb{P}$

Policy $a_1$ $\quad \delta_1 = 1 \quad \delta_2 = 0$

Policy $a_2$ $\quad \delta_2 = 1 \quad \delta_1 = 1$

$$\mathbb{P} = \begin{bmatrix} 0.99 & 0.01 & 0 \\ \delta_1 & 0 & \delta_2 \\ 0 & 0.02 & 0.98 \end{bmatrix}$$

$p = [p_1 \;\; p_2 \;\; 1-p_2-p_3]$

$$\underbrace{[p_1 \;\; p_2 \;\; 1-p_1-p_2]}_{p} = \underbrace{[p_1 \;\; p_2 \;\; 1-p_1-p_2]}_{p} \begin{bmatrix} 0.99 & 0.01 & 0 \\ \delta_1 & 0 & \delta_2 \\ 0 & 0.02 & 0.98 \end{bmatrix} =$$

$$= [0.99p_1 + \delta_1 p_2 \quad 0.01p_1 + 0.02(1-p_1-p_2) \quad \delta_2 p_2 + 0.98(1-p_1-p_2)]$$

Policy $a_1$: (buy new machine)

$$\begin{cases} p_1 = 0.99p_1 + p_2 \\ p_2 = -0.01p_1 - 0.02p_2 + 0.02 \\ 0.02(1-p_1-p_2) = 0 \end{cases} \qquad \begin{cases} p_2 = 0.01p_1 \\ 1.02 \cdot 0.01\, p_1 + 0.01p_1 = 0.02 \\ p_1 + p_2 = 1 \end{cases}$$

$p_2 = 0.01p_1$

$p_1 = \frac{2}{2.02} = \frac{100}{101} \qquad p_2 = \frac{1}{101}$

Policy $a_2$: (repair broken machine)

$$\begin{cases} p_1 = 0.99p_1 \Rightarrow p_1 = 0 \\ p_2 = 0.02(1-p_2) \\ 1-p_2 = p_2 + 0.98(1-p_2) \end{cases} \qquad \begin{aligned} & p_1 = 0 \\ & p_2 = \frac{0.02}{1.02} = \frac{1}{51} \\ & p_3 = 1-p_2 = \frac{p_2}{0.02} = \frac{50}{51} \end{aligned}$$

Relative cost to repair =

$$\frac{p_2(a_2)}{\alpha\, p_2(a_1)} = \frac{1/51}{\alpha\, 1/101} = \frac{101}{51\alpha} < 1 \quad \text{when } \alpha > \frac{101}{51} \approx 1.98$$