Discrete Event Systems

Course code: SSY165

Examination 2013-10-22

Time: 08:30-12:30, Location: H-building

Teacher: Bengt Lennartson, phone 3722

The examination includes 25 points, where grade three requires 10 points, grade four 15 points and grade five 20 points.

The result of this examination will be announced latest on Wednesday *November 6* on the notice board of the division, at the entrance in the south east corner on floor 5 of the ED-building. *Inspection* of the grading is done on Wednesday *November 6* and Thursday *November 7* at 12:30-13:00.

Allowed aids at the examination:

- Standard mathematical tables such as Beta, see also formulas in the end of this examination.
- Pocket calculator.

Good luck!

Department of Signals and Systems
Division of Automatic Control, Automation and Mechatronics
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1

Prove the following set equivalence by equivalence relations based on predicate expressions.

$$A \cap C \subseteq B \cup D \Leftrightarrow (A \cap B \cap C) \cup (A \cap C \cap D) = A \cap C$$

(3 p)

2

Prove that

$$\exists x[p(x) \rightarrow q(x)] \vee \big(\forall x[p(x) \vee q(x)] \wedge \neg\, (\exists x[p(x)] \vee \exists x[q(x)])\big) \Leftrightarrow \forall x[p(x)] \rightarrow \exists x[q(x)]$$

by assuming a universal set Ω with a finite number of arbitrary elements

$$\Omega = \{a_1, a_2, \cdots, a_n\}$$

(3 p)

3

Two parts P_1 and P_2 are manipulated similarly in n steps. This is represented by the formal languages $L(P_i) = a_{i1}a_{i2} \ldots a_{in}$, $i = 1, 2$.

a) Formulate for $n = 3$ corresponding automata for P_1 and P_2, and the synchronous composition $P_1 || P_2$.

(1 p)

b) If the two parts are of the same type and the identity is not necessary to track, the model can be simplified by a Petri net with two tokens, $n + 1$ places and the events $a_1a_2 \ldots a_n$. Formulate this Petri net and the corresponding reachability graph for $n = 3$, and compare the number of states with the synchronized model in a) where the identity of each part is included in the model.

(2 p)

c) Perform part b) but now for an arbitrary n. More specifically, determine the number of states for the first model with part identity and the second simplified Petri net model without part identity for an arbitrary n.

(2 p)

4

Two people named A and B are playing a simple game. A number of sticks are lain out on the ground and the players take alternately one or two sticks. Note that at least one stick must be picked. The player that ends up with the last stick has lost the game. Player A is always the one that starts picking sticks.

a) Model this game by an automaton, with an initial number of n sticks, for $n = 5$. Hint: Introduce a marked state specifying that player A is to win and player B is to loose.

(2 p)

b) Generate a supervisor which guarantees that player A wins the game. Note that the set of uncontrollable events must first be decided.

(2 p)

c) Evaluate if it is possible for player A to always win the game also for $n = 6$ and $n = 7$.

(2 p)

5

The temperature $\theta(t)$ of a heating system is modeled by a first order differential equation

$$T_q\dot{\theta}(t) + \theta(t) = K_q u(t) \qquad q \in \{\text{low, medium, high}\}$$

where the power $u(t)$ is the control signal, and the three time constants and gains (T_q, K_q) correspond to the three different flow levels low, medium, and high. Assume that the flow is measured continuously by a discrete flow sensor $f \in \{$f_low, f_medium, f_high$\}$, and that a switch from one level to any of the other two levels is always possible. Generate a hybrid automaton for this continuous-time system including three discrete flow modes.

(3 p)

6

Two machines M_1 and M_2 are available for producing a specific product. Machine M_1 needs to be repaired more often than M_2, but the cost of producing items is higher for M_2. More specifically, the conditional probability for M_1 moving from its working state to its repair state is $1/3$, while staying in its working state is $2/3$. Corresponding conditional probabilities for M_2 are $1/9$ and $8/9$. A machine being in its repair state is assumed to be immediately repaired.

The cost of repairing both types of machines is 10 units, while the cost being in the working state is 5 units for M_1 and 5α units for M_2, where $\alpha > 1$ due to the higher production cost for machine M_2.

a) Formulate a Markov Decision Process for this problem, where action a_i means that machine M_i is chosen. Write a state transition diagram including the two actions.

(2 p)

b) Decide based on the average cost for which values of α it is more profitable (lower cost) to use M_1 than M_2.

(3 p)

BL 13/027

Solution: Exam DES 2013-10-22

1. Let $E \triangleq A \cap C$ and show that

$E \subseteq B \cup D \Leftrightarrow (E \cap B) \cup (E \cap D) = E$. This is true iff

$$\models (p_E \wedge p_B) \vee (p_E \wedge p_D) \leftrightarrow p_E \Leftrightarrow$$

$$\models \Big(p_E \wedge \underbrace{(p_B \vee p_D)}_{p_F} \to p_E \Big) \wedge \big(p_E \to p_E \wedge p_F \big) \Leftrightarrow$$

$$\models (\neg p_E \vee \neg p_F \vee p_E) \wedge (\neg p_E \vee (p_E \wedge p_F)) \Leftrightarrow$$

$$\models (\underbrace{\neg p_E \vee p_E}_{T} \vee \neg p_F) \wedge (\underbrace{\neg p_E \vee p_E}_{T}) \wedge (\neg p_E \vee p_F) \Leftrightarrow$$

$$\models (T \vee \neg p_F) \wedge (p_E \to p_F) \Leftrightarrow \models p_E \to (p_B \vee p_D)$$

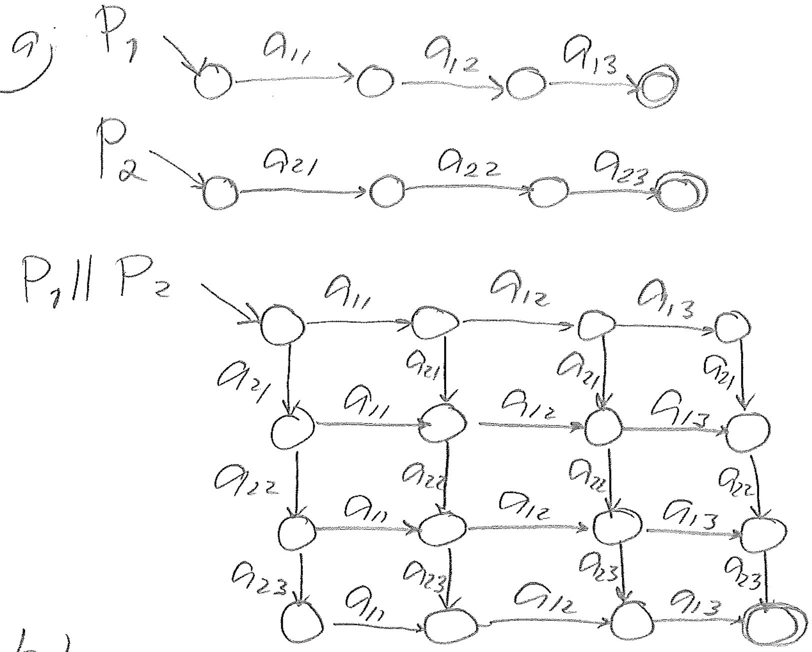
$$\Leftrightarrow \quad E \subseteq B \cup D$$

2.

$$\exists x[p(x) \to q(x)] \vee (\forall x[p(x) \vee q(x)] \wedge \neg(\exists[p(x)] \vee \exists[q(x)]))$$

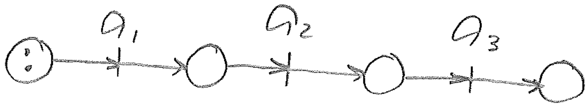
$$\Leftrightarrow \neg p(a_1) \vee q(a_1) \vee \ldots \vee \neg p(a_n) \vee q(a_n) \vee ((p(a_1) \vee q(a_1)) \wedge \ldots \wedge (p(a_n) \vee q(a_n))$$
$$\wedge \neg(p(a_1) \vee \ldots \vee p(a_n) \vee q(a_1) \vee \ldots \vee q(a_n))) \Leftrightarrow \neg(p(a_1) \wedge \ldots \wedge p(a_n))$$
$$\vee (q(a_1) \vee \ldots \vee q(a_n)) \vee ((p(a_1) \vee q(a_1)) \wedge \neg(p(a_1) \vee q(a_1)) \wedge \ldots$$
$$\wedge (p(a_n) \vee q(a_n)) \wedge \neg(p(a_n) \vee q(a_n))) \Leftrightarrow$$
$$\neg \forall x[p(x)] \vee \exists x[q(x)] \vee (\forall x[p(x) \vee q(x)] \wedge \neg \forall x[p(x) \vee q(x)])$$
$$\Leftrightarrow (\forall x[p(x)] \to \exists x[q(x)]) \vee T$$
$$\Leftrightarrow \quad \forall x[p(x)] \to \exists x[q(x)]$$

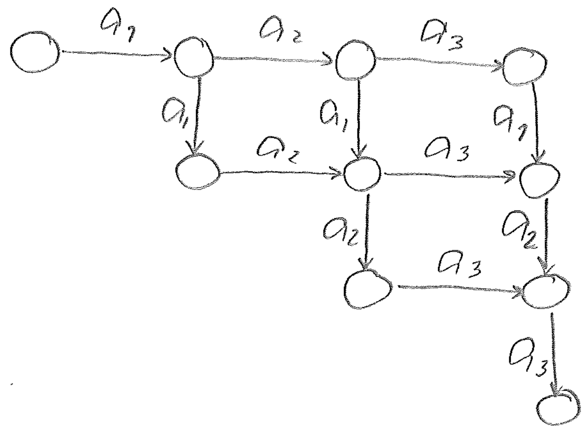
3. a)



Number of states $= (3+1)^2 = 4^2 = 16$

b)





Number of states $= 4+3+2+1 = 10$

c) Number of states with part identity $= (n+1)^2$

Number of states without part identity, corresponding to the PN in b) =

$$= n+1 + n + n-1 + \dots + 1 = \sum_{k=1}^{n+1} k$$

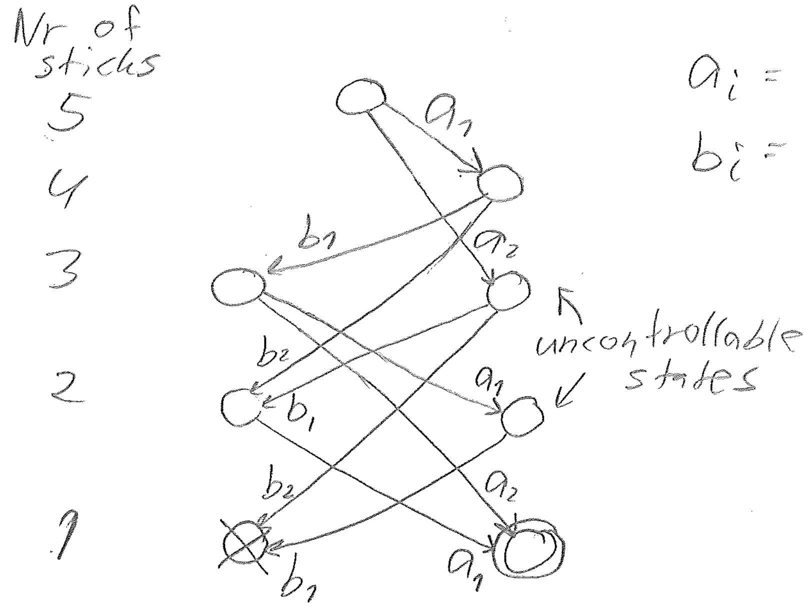
$$= \frac{(n+1)(n+2)}{2} = \frac{(n+1)(n+1+1)}{2} =$$

$$= 0.5(n+1)^2 + 0.5(n+1) \approx 0.5(n+1)^2$$

for large n.

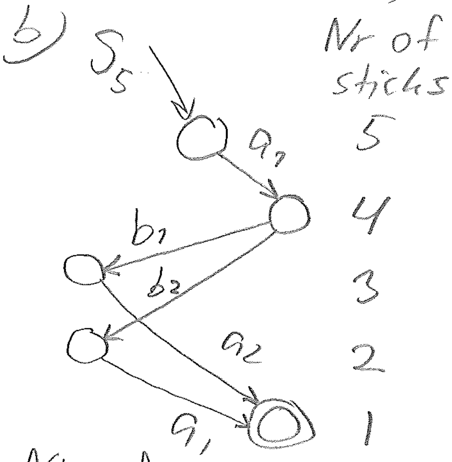
$\therefore$ Avoiding identity in the PN saves $\approx$ half of the state space for large n.

4. a)

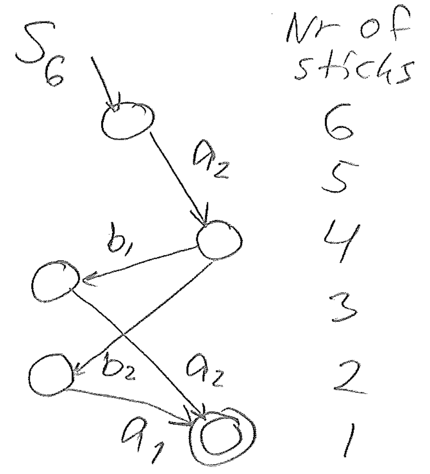


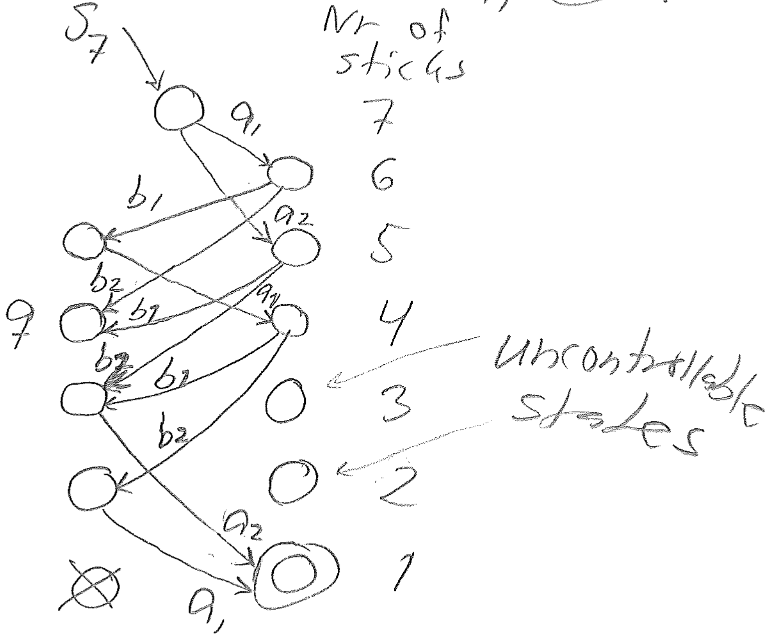
a_i = A takes i stick(s)
b_i = B takes i stick(s) (uncontrollable)

b)



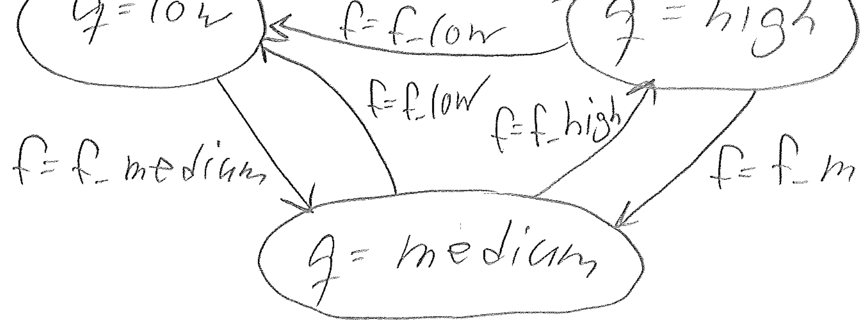
c)





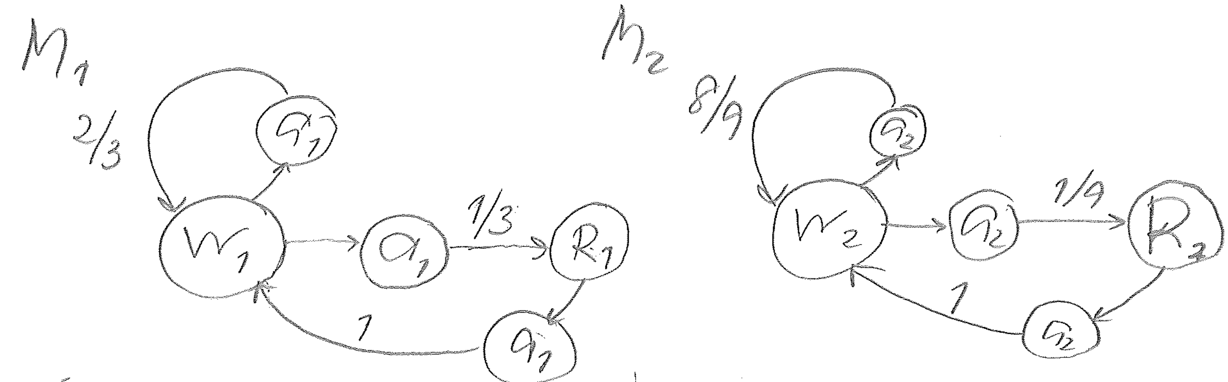
In state q for S_7 A must take one or two sticks ⇒ one of the uncontrollable states will be reached. Hence, A is not guaranteed to win for n=7 but for n=5 and 6.

5.



Continuous model in each state
$T_q \dot{\theta} + \theta = K_q u$

6.



stationary state probability

p_i = working state

$1-p_i$ = repair state

a_1:
$$[p_1 \quad 1-p_1] = [p_1 \quad 1-p_1] \begin{bmatrix} 2/3 & 1/3 \\ 1 & 0 \end{bmatrix}$$

$$\begin{cases} p_1 = 2/3\, p_1 + 1 - p_1 \\ 1-p_1 = 1/3\, p_1 \end{cases} \Rightarrow p_1 = 3/4 \quad 1-p_1 = 1/4$$

$$V_1 = 5p_1 + 10(1-p_1) = \frac{15}{4} + \frac{10}{4} = \frac{25}{4} = 6.25$$

a_2:
$$[p_2 \quad 1-p_2] = [p_2 \quad 1-p_2] \begin{bmatrix} 8/9 & 1/9 \\ 1 & 0 \end{bmatrix}$$

$$\begin{cases} p_2 = 8/9\, p_2 + 1 - p_2 \\ 1-p_2 = 1/9\, p_2 \end{cases} \Rightarrow p_2 = 9/10 \quad 1-p_2 = 1/10$$

$$V_2 = 5\alpha\, p_2 + 10(1-p_2) = 5\alpha \frac{9}{10} + 10 \cdot 1/10 =$$
$$= 9/2\, \alpha + 1$$

$V_1 < V_2$ when $\frac{25}{4} < \frac{9}{2}\alpha + 1$

$$\alpha > \frac{21/4}{9/2} = \frac{7}{6}$$

More profitable to use M_1 for $\alpha > 7/6$