Introduction to Discrete Event Systems

Course code: SSY165, ESS200

Examination 2010-10-21

Time: 14:00-18:00, Location: H-building

Teacher: Bengt Lennartson, phone 3722

The examination includes 25 points, where grade three requires 10 points, grade four 15 points and grade five 20 points.

The result of this examination will be announced latest on Thursday *November 4* on the notice board of the division, at the entrance in the south east corner on floor 5 of the E-building. *Inspection* of the grading is done on Thursday *November 4* and Friday *November 5* at 12:30-13:00.

Allowed aids at the examination:

- Standard mathematical tables such as Beta, see also formulas in the end of this examination.
- Pocket calculator.

Good luck!

Department of Signals and Systems
Division of Automatic Control, Automation and Mechatronics
Chalmers University of Technology

1

Show the following set equivalence by relations based on predicate expressions

$$B \setminus (B \setminus (A \cup D)) \subseteq B \cap C \Leftrightarrow (A \cap B) \cup (B \cap D) \subseteq C$$

(3 p)

2

Two discrete event subsystems P_1 and P_2 are modeled by the following formal languages

$$L(P_1) = \overline{(a(b + c + d)e)^*}$$
$$L(P_2) = \overline{(abe + ace)^*}$$

a) Generate an automaton for the synchronized system $P_1||P_2$, both when $d \in \Sigma_{P_2}$ and $d \notin \Sigma_{P_2}$.

(2 p)

b) Generate the formal language for the synchronized system $P_1||P_2$ and show that

$$L(P_1||P_2) = L(P_1) \cap L(P_2)$$

when $d \in \Sigma_{P_2}$ but not for the case when $d \notin \Sigma_{P_2}$.

(1 p)

c) For the case where $d \notin \Sigma_{P_2}$, modify the language for $L(P_2)$ by including $d \in \Sigma_{P_2}$ such that the synchronized system $P_1||P_2$ gets the same behavior as before, but now also $L(P_1||P_2) = L(P_1) \cap L(P_2)$.

(2 p)

3

Consider a Petri net with two places and two transitions, where the incidence matrix A^+, defining the weights of the arcs between the input transitions and their output places, is

$$A^+ = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$$

the incidence matrix A^-, defining the weights of the arcs between the input places and their output transitions, is

$$A^- = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

and the initial marking vector

$$m_0 = \begin{bmatrix} 1 & 0 \end{bmatrix}^T$$

a) Generate a Petri net based on the given matrix information.

(1 p)

b) Show that the number of tokens in one of the places can increase unboundedly by evaluating the corresponding reachability graph.

(2 p)

c) Add an additional so called control place in the Petri net to avoid that the number of tokens in any of the two places can be more than one. Verify this result by evaluating the reachability graph for the modified Petri net.

(2 p)

4

Generate a controllable and nonblocking supervisor for a plant P where the language

$$L(P) = \overline{a(c + db(c + d)) + cd}$$

and the specification S_p is given by the marked language

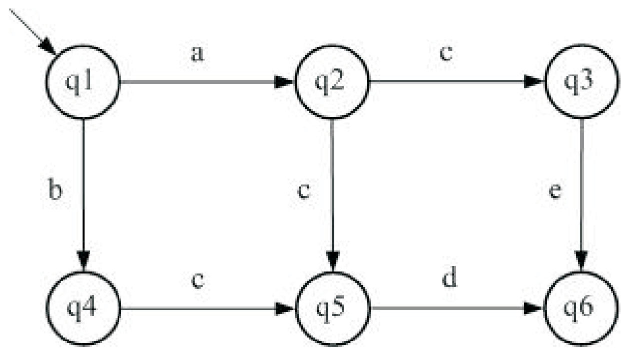
$$L_m(Sp) = a(c + dbc) + cd$$

Assume that the event d is uncontrollable, while the other events are controllable.

(4 p)

5

Generate a maximal permissive supervisor for the plant P given below, where all events are assumed to be controllable. The specification $Sp = P$ with the additional demand that q_3 is a forbidden state and q_6 is the only marked state. Observe that the plant has a nontraditional behavior in the sense that execution of the event c in state q_2 means that the system will reach either the forbidden state q_3 or the acceptable state q_5. The actual choice cannot be controlled by the supervisor, only that the plant is moving to either q_3 or q_5. Give a clear motivation for your suggested supervisor.



(3 p)

6

Consider two cylinders A and B where cylinder A achieves a clamping of a product in its one position, while cylinder B performs a drilling operation in its one position. Cylinder A is equipped with sensors y_0^A and y_1^A measuring its zero and one positions respectively. The piston rod in the cylinder moves forward when $u^A\!:=\!1$ and returns when $u^A\!:=\!0$. Cylinder B has corresponding sensor signals y_0^B and y_1^B and control signal u^B. Assume the following sequence:

1. the cylinders are assumed to be in their zero (home) positions when a new product arrives and an operator push the start button $s = 1$,
2. the piston rod in cylinder A moves out and fixates the product,
3. the piston rod in cylinder B moves out and a drilling operation is performed (the drilling operation starts automatically when $u^B = 1$),
4. the piston rod in cylinder B moves back after the drilling operation when $y_1^B = 1$,
5. the piston rod in cylinder A moves back, which releases the product that is finally removed by the operator.

a) Formulate an extended finite automaton model which generates the control sequence defined above based on the available sensor signals. Include a repeated loop behavior.

(2 p)

b) Generate a state space model

$$x^+ = f(x, y)$$
$$u = g(x, y)$$

where the vector y includes all sensor signals, u desired control signals and x the internal state vector for the controller.

(3 p)

Table 1.1 Equivalence relations.

E_1	$\neg\neg p \Leftrightarrow p$		
E_2	$\neg(p \vee q) \Leftrightarrow \neg p \wedge \neg q$	E_3	$\neg(p \wedge q) \Leftrightarrow \neg p \vee \neg q$
E_4	$p \vee q \Leftrightarrow q \vee p$	E_5	$p \wedge q \Leftrightarrow q \wedge p$
E_6	$p \vee (q \vee r) \Leftrightarrow (p \vee q) \vee r$	E_7	$p \wedge (q \wedge r) \Leftrightarrow (p \wedge q) \wedge r$
E_8	$p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$	E_9	$p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$
E_{10}	$p \vee p \Leftrightarrow p$	E_{11}	$p \wedge p \Leftrightarrow p$
E_{12}	$p \vee \mathbf{F} \Leftrightarrow p$	E_{13}	$p \wedge \mathbf{T} \Leftrightarrow p$
E_{14}	$p \vee \mathbf{T} \Leftrightarrow \mathbf{T}$	E_{15}	$p \wedge \mathbf{F} \Leftrightarrow \mathbf{F}$
E_{16}	$p \vee \neg p \Leftrightarrow \mathbf{T}$	E_{17}	$p \wedge \neg p \Leftrightarrow \mathbf{F}$
E_{18}	$p \vee (p \wedge q) \Leftrightarrow p$	E_{19}	$p \wedge (p \vee q) \Leftrightarrow p$
E_{20}	$p \rightarrow q \Leftrightarrow \neg p \vee q$	E_{21}	$\neg(p \rightarrow q) \Leftrightarrow p \wedge \neg q$

Table 1.2 Implication relations.

I_1	$p \wedge q \Rightarrow p$	I_2	$p \wedge q \Rightarrow q$
I_3	$p \Rightarrow p \vee q$	I_4	$q \Rightarrow p \vee q$
I_5	$\neg p \Rightarrow p \rightarrow q$	I_6	$q \Rightarrow p \rightarrow q$
I_7	$\neg(p \rightarrow q) \Rightarrow p$	I_8	$\neg(p \rightarrow q) \Rightarrow \neg q$

$$A||B = \left\langle Q^A \times Q^B, \Sigma^A \cup \Sigma^B, \delta, \langle q_i^A, q_i^B \rangle, Q_m^A \times Q_m^B, (Q_x^A \times Q^B) \cup (Q^A \times Q_x^B) \right\rangle$$

$$\delta(\langle q^A, q^B \rangle, \sigma) = \begin{cases} \delta^A(q^A, \sigma) \times \delta^B(q^B, \sigma) & \sigma \in \Sigma^A \cap \Sigma^B \\ \delta^A(q^A, \sigma) \times \{q^B\} & \sigma \in \Sigma^A \setminus \Sigma^B \\ \{q^A\} \times \delta^B(q^B, \sigma) & \sigma \in \Sigma^B \setminus \Sigma^A \end{cases}$$

Solution to examination in IDES 101021

1. $B \setminus (B \setminus (A \cup D)) \subseteq B \cap C \Leftrightarrow (A \cap B) \cup (B \cap D) \subseteq C$?

$$p_B(x) \wedge \neg(p_B(x) \wedge \neg(p_A(x) \vee p_D(x))) \rightarrow p_B(x) \wedge p_C(x) \Leftrightarrow$$

$$\neg\big(p_B(x) \wedge (\neg p_B(x) \vee (p_A(x) \vee p_D(x)))\big) \vee (p_B(x) \wedge p_C(x)) \Leftrightarrow$$

$$\neg\Big[\underbrace{(p_B(x) \wedge \neg p_B(x))}_{F} \vee \big(p_B(x) \wedge (p_A(x) \vee p_D(x))\big)\Big] \vee (p_B(x) \wedge p_C(x)) \Leftrightarrow$$

$$\neg p_B(x) \vee \neg(p_A(x) \vee p_D(x)) \vee (p_B(x) \wedge p_C(x)) \Leftrightarrow$$

$$\neg(p_A(x) \vee p_D(x)) \vee \Big(\underbrace{(\neg p_B(x) \vee p_B(x))}_{T} \wedge (\neg p_B(x) \vee p_C(x))\Big) \Leftrightarrow$$

$$\neg(p_A(x) \vee p_D(x)) \vee \neg p_B(x) \vee p_C(x) \Leftrightarrow$$

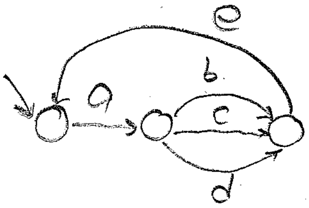
$$\neg\big((p_A(x) \vee p_D(x)) \wedge p_B(x)\big) \vee p_C(x) \Leftrightarrow$$

$$\neg\big((p_A(x) \wedge p_B(x)) \vee (p_D(x) \wedge p_B(x))\big) \vee p_C(x) \Leftrightarrow$$

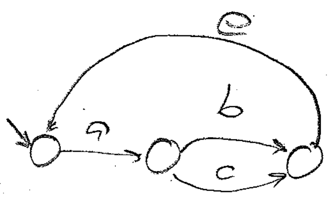
$$(p_A(x) \wedge p_B(x)) \vee (p_B(x) \wedge p_D(x)) \rightarrow p_C(x)$$

The first and last predicate expressions are equivalent and they correspond to the set expressions, which proofs the equivalence on the first line.

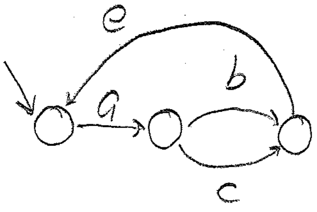
2. a) P_1



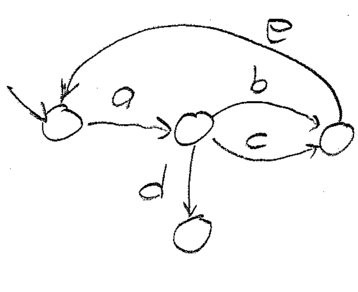
P_2



$P_1 \| P_2 \quad d \in \Sigma_{P_2}$



$P_1 \| P_2 \quad d \notin \Sigma_{P_2}$



b) $d \in \Sigma_{P_2}$ $\quad L(P_1 \| P_2) = L(P_2)$

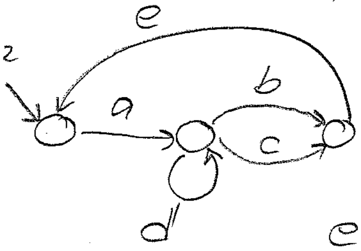
$$L(P_1) \cap L(P_2) = \overline{(abe + ace + ade)^*} \cap \overline{(abe + ace)^*}$$

$$= L(P_2)$$

$d \notin \Sigma_{P_2}$ $\quad L(P_1 \| P_2) = a((b+c)ea)^* d$

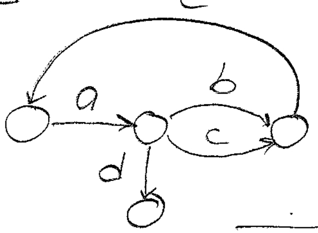
$$\neq L(P_2) = L(P_1) \cap L(P_2)$$

c) P_2



$$L(P_2) = \overline{(a d^* (b+c) e)^*}$$

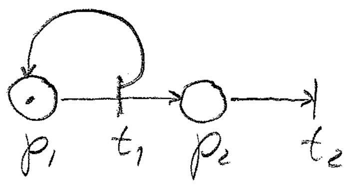
$P_1 \| P_2$



$$L(P_1) \cap L(P_2) = \overline{(abe + ace + ade)^*} \cap \overline{(ad^* be + ad^* ce)^*}$$

$$= a(bea + cea)^* d = L(P_1 \| P_2)$$

3. a)



$$m^+ = m + \underbrace{(A^+ - A^-)}_{A} s_j \quad j = 1,2$$

↑ selection vector

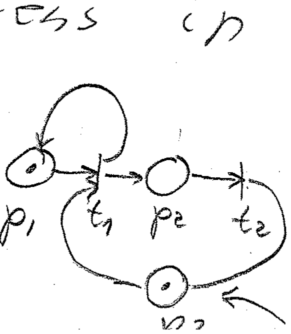
$$A = \begin{bmatrix} 0 & 0 \\ 1 & -1 \end{bmatrix}$$

b)

$(1\ 0)^T$ $\downarrow t_1$ $(1\ 1)^T$ $\downarrow t_2$ $(1\ 2)$ $\downarrow t_3$ $(1\ 3)$ ⋮ $(1\ k-1)^T$ $\downarrow t_k$ $(1\ k)$

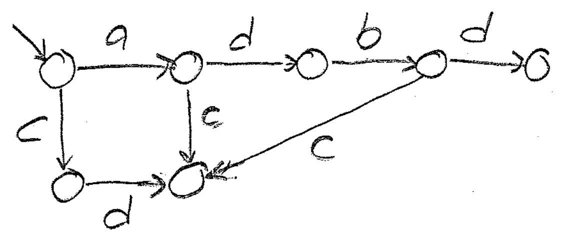
Infinite number of t_1 transitions and a limited number of t_2 transitions $\Rightarrow$ infinite number of tokens in p_1.

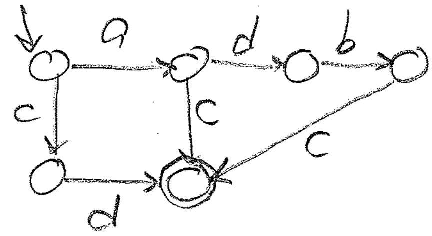
c)



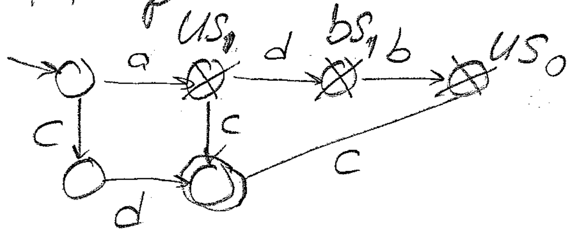
← control place

$(1\ 0\ 1)^T$ $\downarrow t_1$ $(1\ 1\ 0)^T$

4. P 

S_P 

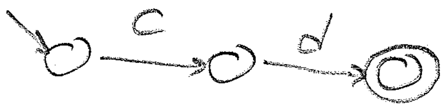
$S_0 = P \| S_P$



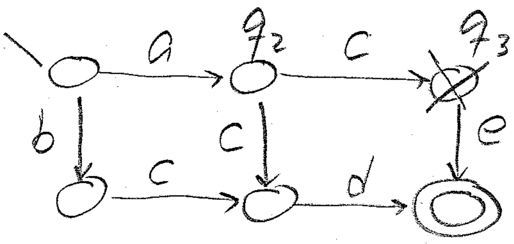
us_k = uncontrollable state in iteration k

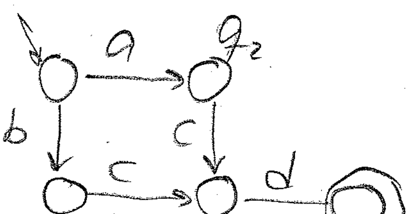
bs_k = blocking state in iteration k

Resulting supervisor

S 

$L(S) = \overline{cd}$

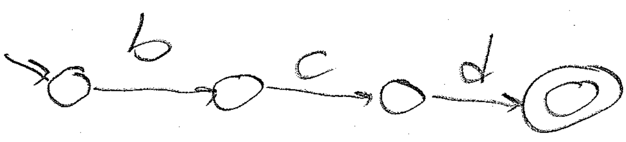
5. S_P 

$S_0 = P \| S_P$ 

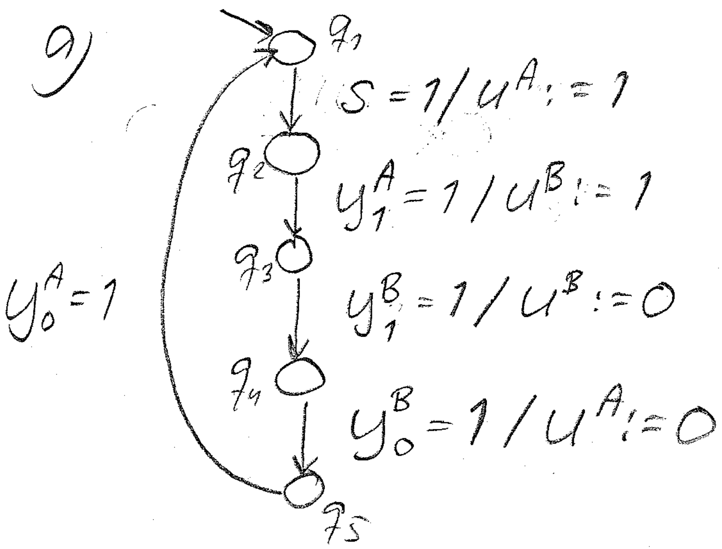
When the plant is in state q_2 and c is executed the plant may transfer to the forbidden state q_3.

Thus state q_2 must be excluded from the supervisor although the event c is controllable.

Resulting supervisor is then

S 

6. a)



$q = q_i \leftrightarrow x_i = 1$

b)

$$x_1^+ = (x_5 \wedge y_0^A) \vee (x_1 \wedge \neg S)$$
$$x_2^+ = (x_1 \wedge S) \vee (x_2 \wedge \neg y_1^A)$$
$$x_3^+ = (x_2 \wedge y_1^A) \vee (x_3 \wedge \neg y_1^B)$$
$$x_4^+ = (x_3 \wedge y_1^B) \vee (x_4 \wedge \neg y_0^B)$$
$$x_5^+ = (x_4 \wedge y_0^B) \vee (x_5 \wedge \neg y_0^A)$$

$$u^A = x_2 \vee x_3 \vee x_4$$
$$u^B = x_3$$

$$y = \begin{bmatrix} S \\ y_0^A \\ y_1^A \\ y_0^B \\ y_1^B \end{bmatrix} \quad u = \begin{bmatrix} u^A \\ u^B \end{bmatrix} \quad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}$$