## Chalmers University of Technology

Department of Signals and Systems

# SSY155 Applied Mechatronics <br> Examination date 100312 

Time and place: $\quad 14: 00-18: 00$ at V building
Teacher: Esteban Gelso, tel 031-772 1855

## Allowed material during the exam:

- Mathematical handbook of your choice.
- Calculator without plotting functionality.
- 1 handwritten A4-page with information of your choice.
- Scientific papers:
- Roos, F., Johansson, H., and Wikander, J., Optimal selection of motor and gearhead in mechatronic application, Mechatronics, (63-72), vol 16, 2006.
- Glad, T., and Ljung, L., Chapter 14 from Control Theory, Multivariable and Nonlinear Methods.

The exam consists of 5 exercises of a total of 50 points. Nominal grading according to 23/30/40 points, you need 23 points to pass the course with grade 3,30 points to pass with grade 4 and 40 to pass the course with grade 5 . Solutions and answers should be written in English and be unambiguous and well motivated, but preferably short and concise.

Those who received at least 3 bonus points from the hand in assignments during the course 2010 can skip the first problem in the exam, and you receive 10 points for that problem for free. You should have received an email prior to the exam with your number of bonus points.

The fifth problem has two different alternatives. For this problem, you can choose freely one of them. If you choose to solve both alternatives, you will receive the points of that one in which you received most points.

Results are mailed out latest March 24. You may check your grading of your exam on March 24 at 12.30-13.00 at the Department of Signals and Systems.

1. Motor and gear ratio is to be chosen in a mechatronic construction. There are two motors to choose between. Gear ratio can be chosen freely. The motors have the following specifications: $\omega_{\text {max }}^{1}=4800 \mathrm{rpm}, T_{\text {peak }}^{1}=4.8 \mathrm{Nm}, T_{\text {rated }}^{1}=1.2 \mathrm{Nm}$, $J_{m}^{1}=4.5 \mathrm{kgcm}^{2}$ and $\omega_{\text {max }}^{2}=4200 \mathrm{rpm}, T_{\text {peak }}^{2}=7.2 \mathrm{Nm}, T_{\text {rated }}^{2}=1.8 \mathrm{Nm}$, $J_{m}^{2}=5.8 \mathrm{kgcm}^{2}$. Assume an ideal gearbox.
From the product specifications of required speed cycle the following integrals are formed over one work cycle of the construction

$$
\begin{aligned}
& k_{1}=\frac{1}{T} \int_{0}^{T} \ddot{\theta}_{l}^{2}(t) d t=2500 \mathrm{~s}^{-4} \\
& k_{2}=\frac{1}{T} \int_{0}^{T} T_{l}^{2}(t) d t=400 \mathrm{~N}^{2} \mathrm{~m}^{2} \\
& k_{3}=\frac{1}{T} \int_{0}^{T} \ddot{\theta}_{l}(t) T_{l}(t) d t=1000 \mathrm{Nm} / \mathrm{s}^{2}
\end{aligned}
$$

For the work cycle the maximum speed and torque have also been recorded: $T_{l}^{\text {peak }}=30 \mathrm{Nm}$ and $\omega_{l}^{\max }=150 \mathrm{rpm}$.
Select motor and gear ratio so that the RMS torque is minimized.
2. (a) A 50 V , DC shunt motor drives a constant-torque load at a speed of 1200 rpm . The armature and field resistances are $2 \Omega$ and $100 \Omega$, respectively. The motor draws a line current of 15 A at the given load.
(a.1) Calculate the resistance that should be added to the armature circuit to reduce the speed by 25 percent.
[5p]
(a.2) Assume that except the heat losses in the armature and field resistances, the motor has additional rotational losses of 80 W . Calculate the efficiency of the motor without and with the added resistance.
[2p]
(a.3) In part (a.1) you reduce the speed by adding a resistance to the armature circuit. Can you propose an alternative way to reduce the speed of the motor with less heat losses?
(b) Briefly compare features of a DC motor with the shunt-wound and serieswound configurations for the armature and field windings. Sketch the steadystate torque-speed curves for these types of winding arrangements.
[2p]
3. (a) What is the reason of using freewheeling diodes, sometimes called flyback diodes, in motor control applications? These diodes are generally connected in parallel with power transistors.
[2p]
(b) Describe the main tasks in a diagnostic system in a few words. How can faults be classified depending on the temporal aspects? Give some examples. [2p]
(c) Compare a gear reducer mechanism against a belt and pulley mechanism by giving at least two advantages and two disadvantages.
(d) What is energy harvesting?
(e) How does a microcontroller differ from a microprocessor?
4. A strain gage load cell consists of four identical strain gages, forming a Wheatstone bridge, that are mounted on a rod that has square cross-section. One opposite pair of strain gages is mounted axially and the other pair is mounted in the traverse direction.
(a) When the load cell is subject to a compressive force, the compressive gauges, i.e. $R_{1}$ and $R_{4}$ in the figure, suffer a strain of $-1.0 * 10^{-5}$ and the tensile gauges, i.e. $R_{2}$ and $R_{3}$, suffer a strain of $+0.3 * 10^{-5}$. The gauges have a gauge factor of 2.1 and a resistance of $120 \Omega$. The supply voltage voltage for the bridge, $V_{i}$, is 10 V .
If the amplifier circuit is not considered in this item, what will be the voltage output from the bridge $V_{B}$ ?

[4p]
(b) If the output voltage from the bridge calculated above is amplified by a differential operational amplifier circuit, what ratio of $R_{5}$ and $R_{6}$ would give an output $V_{O}$ of 100 mV ? Assume that $R_{5}, R_{6} \gg R_{1}, R_{2}, R_{3}, R_{4}$.
[4p]
(c) The output signal of the signal conditioning stage is to be fed into an analogue-to-digital converter. If the ADC has a range of $0 \mathrm{~V}-5 \mathrm{~V}$, determine the number of bits required to obtain a resolution of 5 mV .
[2p]
5.
(ALTERNATIVE 1) The dynamics of the steering system of an autonomous vehicle is approximated by the following transfer function:

$$
T(s)=\frac{1}{(s+1)(s+2)}
$$

Design a PID controller,

$$
F_{P I D}(s)=K_{p}+\frac{K_{i}}{s}+K_{d} s
$$

so that the following specifications for the close loop system are met:

- The closed loop system should be stable, and the poles should be real and placed in $-5,-10$, and -10 .
[10p]
(ALTERNATIVE 2) Determine the largest $K$ which preserves stability of the system.
If $K=3 K_{\max }$, find approximately the amplitude and the frequency of the self-sustained oscillations. (You will have to solve numerically or graphically an equation to find the amplitude of the self-sustained oscillations.)



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## Solutions

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1. $N_{\max }^{i}=\omega_{\max }^{i} / \omega_{l}^{\max }$, gives $N_{\max }^{1}=32$ and $N_{\max }^{2}=28 . N_{\min }=T_{l}^{\text {peak }} / T_{\text {peak }}$, gives $N_{\text {min }}^{1}=6.25$ and $N_{\text {min }}^{2}=4.2$. Within this interval $T_{\mathrm{RMS}}^{m}$ should be minimized with respect to $N$. That is, minimize

$$
T_{\mathrm{RMS}}^{m}(N)^{2}=J_{m}^{2} N^{2} k_{1}+\frac{k_{2}}{N^{2}}+2 J_{m} k_{3}
$$

Taking the derivative and solving this gives minimum at $N^{1}=29.8$, and $N^{2}=26.2$. These ratios are within the limits for each motor and one obtains $T_{\text {RMS }}^{1}(29.8)=1.34 \mathrm{Nm}$ and $T_{\text {RMS }}^{2}(26.2)=1.52 \mathrm{Nm}$. Motor 1 has the lowest RMS torque but it is above its rated torque. Hence, motor 2 is chosen with $N^{2}=26.2$.
2. (a.1) $I_{f}=V_{t} / R_{f}=50 / 100=0.5 \mathrm{~A}$
$I_{a}=I-I_{f}=15-0.5=14.5 \mathrm{~A}$
$R_{a d d}=\left(V_{t}-0.75 *(50-14.5 * 2)\right) / 14.5-2=0.3621 \Omega$
(a.2) Loss $1=I_{f}^{2} * R_{f}+I_{a}^{2} * R_{a}+$ losses $=525.5 \mathrm{~W}$
$\eta_{1}=\left(P_{\text {in }}-\operatorname{Loss} 1\right) / P_{\text {in }} * 100=29.9 \%$

Loss2 $=I_{f}^{2} * R_{f}+I_{a}^{2} *\left(R_{a}+R_{\text {add }}\right)+$ losses $=601.6 \mathrm{~W}$
$\eta_{2}=\left(P_{\text {in }}-\right.$ Loss 2$) / P_{\text {in }} * 100=19.9 \%$
(a.3) A better solution would be to reduce voltage with a dc/dc converter to slow down the motor.
(b) Shunt motors exhibit very nearly constant speed over a large range of loading. Series motors have very high starting torques. As load is removed from a series motor, the speed will increase sharply. For this reason, series-wound motors must have a load connected to prevent damage from high speed conditions.
3. (a) A freewheeling diode, sometimes called a flyback diode, is a diode used for voltage surge protection due to inductive loads.
(b) Fault detection (finding out that something is going wrong in the monitored process), fault isolation (determining the exact location that is the cause for the deviations from nominal or expected behavior), and fault identification (determining the magnitude of the fault). Abrupt faults: sensor bias; incipient faults: drifts; intermittent faults: intermittent short circuit.
(c) Belt drives have the advantage that the length of the belt can easily be adjusted to suit a wide range of shaft-to-shaft distances and the system is automatically protected against overload. Over small distances gears are to be preferred. Gear reducer mechanisms are usually used in more precision controlled applications.
(d) Energy harvesting is the process by which energy is derived from external sources, captured, and stored.
(e) A microcontroller is an integrated microprocessor chip with many I/O interfaces such as ADC, DAC, PWM, digital I/O, and the communication bus.
4. (a) $\delta R_{1}=\delta R_{4}=G \epsilon R=-2.1 * 1.0 * 10^{-5} * 120=-2.52 * 10^{-3} \Omega$
$\delta R_{2}=\delta R_{3}=G \epsilon R=2.1 * 0.3 * 10^{-5} * 120=7.56 * 10^{-4} \Omega$
$V_{B}=V_{i}\left(\frac{R_{3}}{R_{3}+R_{4}}-\frac{R_{1}}{R_{1}+R_{2}}\right)=V_{i}\left(\frac{R_{2} R_{3}-R_{1} R_{4}}{\left(R_{1}+R_{2}\right)\left(R_{3}+R_{4}\right)}\right)$
$V_{B}=V_{i}\left(\frac{\left(R_{2}+\delta R_{2}\right)\left(R_{3}+\delta R_{3}\right)-\left(R_{1}+\delta R_{1}\right)\left(R_{4}+\delta R_{4}\right)}{\left(R_{1}+\delta R_{1}+R_{2}+\delta R_{2}\right)\left(R_{3}+\delta R_{3}+R_{4}+\delta R_{4}\right)}\right)$
The output is thus $V_{B}=1.37 * 10^{-4} V$
(b) If $R_{5} \gg R_{1} / / R_{2}$ and $R_{5} \gg R_{3} / / R_{4}$ then, the gain of the amplifier circuit is
$\frac{V_{0}}{V_{B}}=\frac{R_{6}}{R_{5}}\left(1+\frac{39}{1}\right)$
If $V_{O}=0.100 V$ when $V_{B}=1.37 * 10^{-4} V$ then,
$\frac{R_{6}}{R_{5}}=18.3$
We can choose, for example, $R_{5}=3 k \Omega$ and $R_{6}=55 k \Omega$.
(c) $\mathrm{Q}=$ resolution and $\mathrm{R}=$ range, $Q=R /\left(2^{n}-1\right)$ then $2^{n}=R / Q+1$
$n=\log (R / Q+1) / \log (2)=9.97$
$n=10 \mathrm{bits}$
5.
(ALTERNATIVE 1)

$$
T(s) * F_{P D}(s)=\frac{K_{d} s^{2} K_{p} s+K_{i}}{s^{3}+3 s^{2}+2 s}
$$

The characteristic equation for the closed loop system becomes

$$
\begin{array}{r}
s^{3}+s^{2}\left(3+K_{d}\right)+s\left(K_{p}+2\right)+K_{i}=0 \\
(s+a)(s+b)(s+c)=s^{3}+s^{2}(a+b+c)+s(a b+a c+b c)+a b c
\end{array}
$$

$a=5, b=10, c=10$, hence

$$
\begin{array}{r}
K_{d}=22 \\
K_{p}=198 \\
K_{i}=500
\end{array}
$$

(ALTERNATIVE 2) The describing function of the saturation is real, so a good starting point is to find at which frequency the transfer function cross the negative real axis, and with which amplitude.

$$
i \omega\left(0.01(i \omega)^{2}+0.1 i \omega+1\right)=-0.1 \omega^{2}+i \omega\left(1-0.01 \omega^{2}\right)
$$

The imaginary part vanish when $1-0.01 \omega^{2}=0$ which give $\omega=10$.
If $\omega=10$ then $G(i \omega)=-K / 10$
$Y_{f}(C)=12 * 2 / \pi\left(\arcsin (1 / C)+1 / C \sqrt{1-1 / C^{2}}\right)$ when $C>1$

$$
-\frac{K}{10} \frac{12 * 2}{\pi}\left(\arcsin \left(\frac{1}{C}\right)+\frac{1}{C} \sqrt{1-\left(\frac{1}{C}\right)^{2}}\right)=-1
$$

To avoid self-oscillations $K$ should be smaller than $K_{\max }=10 / 12=5 / 6$. If $K=3 K_{\max }=2.5$, the Nyquist curve passes through -0.25 and we need to find at which $C$ the describing function satisfies $G(i 10)=-1 / Y_{f}(C)=$ $-0.25$

| $C$ | $-1 / Y_{f}(C)$ |
| :---: | :---: |
| 1 | -0.083 |
| 2.5 | -0.168 |
| 3.75 | -0.248 |
| 5 | -0.329 |
| 10 | -0.655 |

which gives approximately $C=3.75$

