## Chalmers University of Technology <br> Department of Signals and Systems

## SSY155 Applied Mechatronics

## Examination date 100114

Time: 14:00-18:00

Teacher: Esteban Gelso, tel 031-772 1855

## Allowed material during the exam:

- Mathematical handbook of your choice.
- Calculator without plotting functionality
- 1 handwritten A4-page with information of your choice
- Scientific papers:
- Nordin, M., Gutman, P.-O., Controlling mechanical systems with backlash - a survey, Automatica, (1633-1649), 38, 2002.
- Roos, F., Johansson, H., and Wikander, J., Optimal selection of motor and gearhead in mechatronic application, Mechatronics, (63-72), vol 16, 2006.
- Glad, T., and Ljung, L., Chapter 14 from Control Theory, Multivariable and Nonlinear Methods

The exam consists of 5 exercises of a total of 50 points. Nominal grading according to 23/30/40 points, you need 23 points to pass the course with grade 3,30 points to pass with grade 4 and 40 to pass the course with grade 5 . Solutions and answers should be written in English and be unambiguous and well motivated, but preferably short and concise.

Results are mailed out latest two weeks after the exam.

You may check your grading of your exam. Time and place will be mailed out in connection with the examination result.

1. Consider a permanent magnet DC motor. The armature resistance is measured to be $R_{a}=0.5 \Omega$. When $V_{t}=120 \mathrm{~V}$ is applied to the motor, it reaches 1200 rpm steady-state speed and draws 40 A. Determine
a the back emf voltage,
b resistance losses,
c power delivered to the armature,
d and torque generated.
2. Consider a ball screw motion conversion mechanism with a pitch of $p=10 \mathrm{rev} / \mathrm{cm}$ which is used to position a table carrying a workpiece. Note that the the movement of the table is assumed to be horizontal. The mass of the table and workpiece is $m=500 \mathrm{~kg}$, and the resistance force of the load is $F_{r}=500 \mathrm{~N}$. Determine the reflected rotary inertia and torque seen by a motor at the input shaft of the ball screw.
3. a Draw the model of an op_amp and describe the idealized assumptions on it. [2p]
b What is a Programable Logic Computer (PLC)?
c For a mechatronic position system a PD controller is being used. A position sensor is available and for the D-part one can choose between including a speed sensor or taking the derivative of the position signal. What is the advantage/disadvantage with these two possibilities?
d Describe in a few words why it normally is advantageous in mechatronic applications to use a high speed motor in connection with gears reducing the velocity.
4. Consider the following system, controlled with a relay

and with $K=1$. Relays, often gives oscillations in the system. One way to lower the oscillations is to introduce a dead-zone in the relay.
a Determine frequency and amplitude of the oscillation in the system.
b Determine the necessary size of the dead-zone to decrease the amplitude of the oscillation by a factor 2 .
5. The temperature of a greenhouse is measured for control purposes. The temperature varies between $0^{\circ} \mathrm{C}$ and $50^{\circ} \mathrm{C}$ and is measured by using a resistance temperature detector (RTD) sensor, type PT100. The following circuit is used in the signal conditioning stage to amplify the signal of the sensor. Thus, the voltage at the output Vo varies between 0 V and $10 \mathrm{~V}: 0 \mathrm{~V}$ corresponds to $0^{\circ} \mathrm{C}$ and 10 V to $50^{\circ} \mathrm{C}$.
Determine the values of the resistances $R 2$ and $R 3$ if:
$R\left(\right.$ PT100 @ $\left.0^{\circ} C\right)=100 \Omega$
$R\left(\right.$ PT100 @ $\left.50^{\circ} \mathrm{C}\right)=138.5 \Omega$

[10p]
6. a The voltage relation is

$$
U=L \frac{d i(t)}{d t}+R i(t)+U_{b e m f}
$$

At steady-state given, this becomes

$$
120 V=0+0.5 \cdot 40 \Omega A+U_{b e m f}
$$

and $U_{\text {bemf }}=100 \mathrm{~V}$.
b

$$
R \cdot i^{2}=0.5 \cdot 40^{2} \Omega A^{2}=800 \mathrm{~W}
$$

c

$$
P_{m}=U_{\text {bemf }} i=100 \mathrm{~V} \cdot 40 \mathrm{~A}=4 \mathrm{~kW}
$$

d

$$
\omega \cdot T=P_{m} \Rightarrow T=\frac{4000 \mathrm{~W}}{1200 \cdot 2 \pi / 60 s^{-1}}=31.8 \mathrm{Nm}
$$

2. Distance to rotation: $x \cdot p=\omega$. Force to torque: $F / p=T$. This gives the following relation for the reflected inertia

$$
J_{e f f} \dot{\omega}=J_{e f f} \ddot{x} p=T=F_{r} / p
$$

Compared to $\ddot{x} m=F$ this gives $J_{\text {eff }} p^{2}=m$ and, hence, $J_{\text {eff }}=500 /\left(102 \pi 10^{2}\right)^{2} \mathrm{~kg} \mathrm{~m}^{2}=$ $1.27 \cdot 10^{-5} \mathrm{~kg} \mathrm{~m}^{2}$. The torque on the input shaft becomes $T_{\text {eff }}=F_{r} / p=500 /\left(102 \pi 10^{2}\right)=$ $8.0 \cdot 10^{-2} \mathrm{Nm}$. If the movements is assumed to be vertical: $T_{\text {eff }}=\left(F_{r}+m g\right) / p=$ $(500+5009.81) /\left(102 \pi 10^{2}\right)=0.87 N m$
3. a $\omega=\sqrt{2}$ and $C=2 / 3 \pi$.
b The Nyquist curve of the linear part of the system is not changed but the describing function. The amplitude is half if the following equation hold

$$
\frac{4}{\pi C / 2} \sqrt{1-\frac{D^{2}}{(C / 2)^{2}}}=\frac{4}{\pi C}
$$

where $D$ is the dead-zone. The solution is $D=\sqrt{3} C / 4=1 /(2 \pi \sqrt{3})$.
4. a Open-loop gain $\infty$, Input impedance $\infty$, Output impedance 0 , infinite bandwidth.
b A PLC is a robust industrial computer specially designed for control and automation tasks.
c A speed sensor brings an extra cost of the sensor. Taking the derivative of the position signal can be noise sensitive.
d To obtain the speeds and torques typically required in mechatronic applications, it is cheaper (in cost and weight) to use a smaller high-speed motor and to reduce speed (and increase torque) with gears.

5.

$$
\begin{gather*}
V_{o}=i_{1}\left(R_{3}+R_{4}\right)=V_{1} \frac{R 3+R 4}{R_{4}}  \tag{1}\\
V_{1}=V_{2}-i_{2} R_{5}=V_{2} \frac{R_{4}+R_{5}}{R_{4}}-V_{4} \frac{R_{5}}{R_{4}}  \tag{2}\\
V_{2}=V_{3} \frac{R_{5}}{R_{4}+R_{5}} \tag{3}
\end{gather*}
$$

Replace (3) in (2)

$$
\begin{equation*}
V_{1}=\left(V_{3}-V_{4}\right) \frac{R_{5}}{R_{4}} \tag{4}
\end{equation*}
$$

Find expression for $V_{3}$

$$
\begin{gather*}
R_{6}=\frac{R_{p}\left(R_{4}+R_{5}\right)}{R_{p}+R_{4}+R_{5}}  \tag{5}\\
V_{3}=V_{z} \frac{R_{6}}{R_{6}+R_{4}}=V_{z} \frac{R_{p}\left(R_{4}+R_{5}\right)}{R_{7}}  \tag{6}\\
R_{7}=\left(R_{4}+R_{5}\right)\left(R_{p}+R_{4}\right)+R_{p} R_{4} \tag{7}
\end{gather*}
$$

Find expression for $V_{4}$

$$
\begin{gather*}
V_{z}=V_{4}+\left(i_{2}+i_{3}\right) R_{4}=V_{4}+\left(\frac{V_{4}-V_{2}}{R_{4}}+\frac{V_{4}}{R_{2}}\right) R_{4} \Rightarrow  \tag{8}\\
V_{4}=V_{z} \frac{R_{2}}{2 R_{2}+R_{4}} \frac{R_{7}+R_{5} R_{p}}{R_{7}} \tag{9}
\end{gather*}
$$

Replace (9) and (6) in (4) and then (4) in (1)

$$
\begin{equation*}
V_{o}=V_{z} \frac{\left(R_{3}+R_{4}\right)\left(R_{p}-R_{2}\right)\left(R_{4}+R_{5}\right) R_{5}}{R_{4} R_{7}\left(R_{4}+2 R_{2}\right)} \tag{10}
\end{equation*}
$$

(a) $V_{o}=0 \Leftrightarrow R_{2}=R_{p}=100 \Omega$
(b) $V_{o}=10 \mathrm{~V}, R_{p}=138.5 \Omega$ and from (10), $R_{3} \approx 48 \mathrm{k} \Omega$

