Solutions to Examination
SSY130 Applied Signal Processing

14:00-18:00, Jan. 14, 2015

1. In the following subproblems you will get 1 point if the answer is correct and 1 point if the answer is correctly motivated. All frequency scales are relative to the sampling frequency, i.e. 1 corresponds to the sampling frequency.

   (a) The magnitude of the DTFT of two FIR filters designed with the Parks-McClellan optimal equiripple FIR filter design method are shown in Figure 1 (top left). Which of the two filters have the longest length. (2pt)

   (b) Two high-pass filters of the same length are designed using the window method. Filter A is designed using the window function A and filter B is designed with window function B. The DTFT of the two window functions are shown in Figure 1 (top right).

      i. Which of the two filters will have the best stop-band attenuation? (2pt)

      ii. Which of the two filters will have the largest transition band? (2pt)

   (c) The magnitude of the DTFT of the impulse response of a filter is shown in Figure 1 (bottom left.

      i. Describe the type of filter (HP,LP,BP or BS) (2pt)

      ii. What is/are the crossover frequency/frequencies in Hertz if we assume the sample rate of the filter is 20 kHz. (2pt)

   Solution:

   (a) A longer filter means a more flexible filter. A flexible filter can be made to have better performance, for example better stop band attenuation. ⇒ Filter II is the longest.

   (b) i. The frequency function for this design is the result of the frequency domain convolution between the ideal LP-filter and the DTFT of the window function. If the window has small side lobes ⇒ the filter will have a high stop band attenuation. Since Window A has the lowest side lobes ⇒ Filter A will have the best stop-band attenuation.

      ii. Again due to the convolution the width of the transition band is proportional to the width of the main lobe of the window function. ⇒ Filter A will have the largest transition band.

   (c) i. Since the DTFT is plotted in relative frequency where 1=sampling frequency and 0.5 is Nyquist frequency. Hence filter is a High-Pass filter (HP).

      ii. The crossover frequency for the filter is at relative frequency 0.4 which in Hertz is $f_c = 0.4 * 20$ kHz $= 8$ kHz.

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2. This problem is about signal interpolation and it's implementation.

   (a) Describe the two basic signal processing operations found in a linear signal interpolation function and what their purpose are. (3pt)

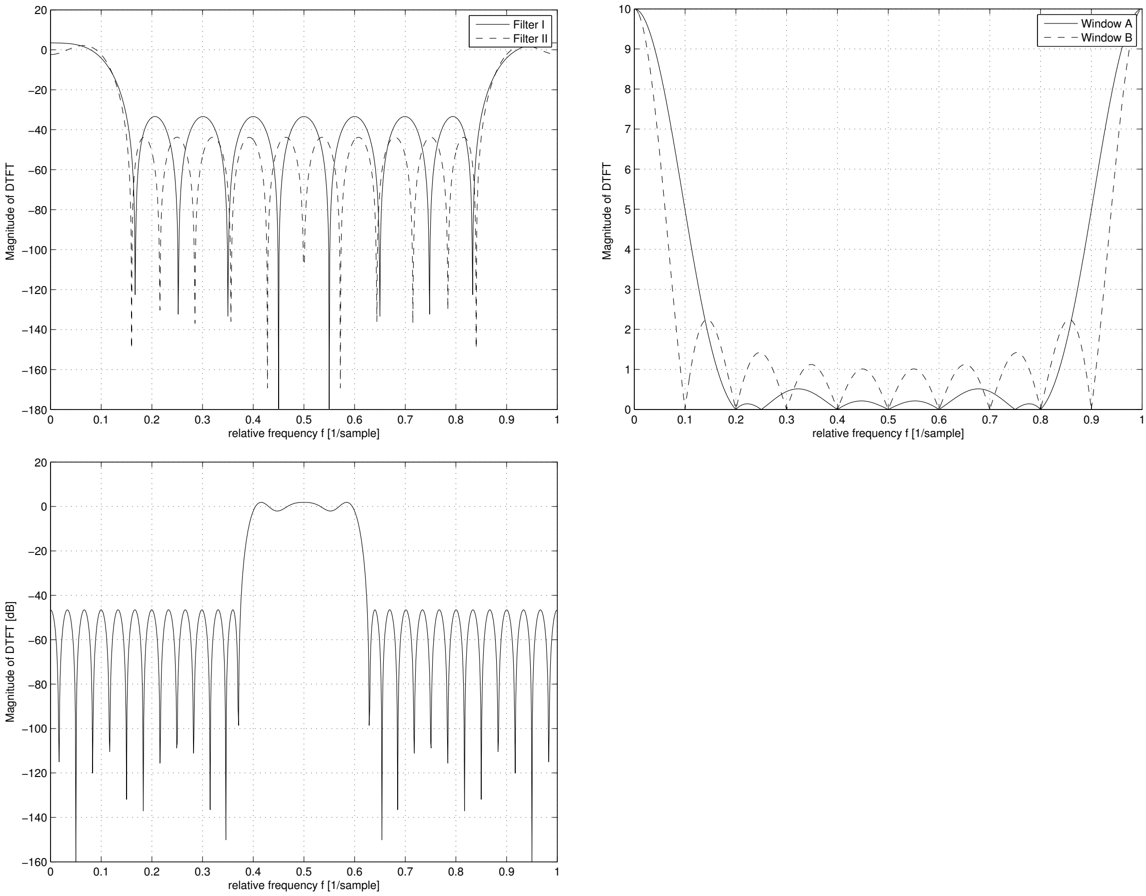

Figure 1: Magnitude DTFT of two FIR filters (top left). Magnitude DTFT of two window functions (top right). Magnitude DTFT of the impulse response of a filter (bottom left).

(b) Assume the original real signal has a sample rate of 1kHz. Describe an interpolation design which will change the sample rate to 5 kHz but keep the original signal bandwidth. Specify the edge frequencies of the passband of the filter involved. (3pt)

(c) Describe what a polyphase filter representation is and how it can be applied to signal interpolation. (3pt)

Solution:

(a) A signal interpolator which increases the sample rate a factor L can be constructed by up-sampling the signal, (i.e. inserting $L-1$ zero samples between the original samples) and as a second step apply a low pass filter to remove the $L-1$ images in the spectrum caused by the up-sampling operation.

(b) To change from 1 to 5 kHz we need a factor five ($L = 5$) upsampling operation. The final low-pass filter should retain the original signal information and hence should have a cut off frequency at 500 Hz which is the Nyquist frequency of the original signal.

(c) A polyphase filter representation is the decomposition of an orignal FIR filter $H(z)$ into a sum of polyphase filter components A_i such that

$$H(z) = \sum_{k=0}^{M-1} h(k)z^{-k} = \sum_{i=0}^{L-1} z^{-i} A_i(z^L)$$

where

$$A_i(z) = \sum_{k=0}^{P-1} a_{k,i} z^{-k}$$

and $P = \text{ceil}(M/L)$. With an interpolation filter in polyphase form we can use the noble identity to change the order of the up-sampler and the L filtering blocks. The filtering operations are then performed at the low rate side. This means that the number of multiplications needed is a factor $1/L$ lower as compared to filtering after the upsampling step.

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3. In telephone applications such as, conference telephones, speaker phones or mobile handsfree operation, a common feature is that the loudspeaker and the microphone is located in the same acoustic environment. The microphone of the receiver not only picks upp the voice of the person localized in the same room as the telephone (local caller) but *also* the sound originating from the remote caller. If this effect is not mitigated (reduced) by some technique the remote caller will hear his own voice with some delay depending on the transmission delay to and from the local caller and it often sounds like an echo.

(a) Construct a signal block diagram explaining the setup described above. Use the following notation:

$s_r(n)$ is the signal from the remote caller at the remote site

$G(\omega)$ is the frequency function modeling the transmission path between the local site and the remote site. For simplicity we assume it is symmetric, i.e. the frequency function is equal in both directions.

$s_l(n)$ is the signal from the local caller

$H(\omega)$ is the frequency function between the local loudspeaker and the local microphone

$z_r(n)$ is the signal the remote caller hears at the remote site. (5pt)

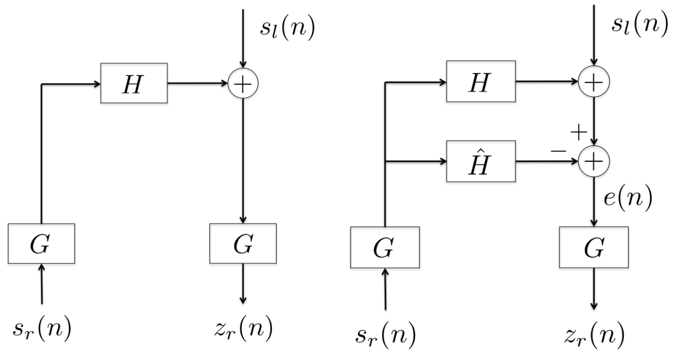
(b) To mitigate the echo effect add an adaptive filter to the local site. Make a new block diagram which includes the frequency function of the adaptive filter and describe which signals are used to update the filter. (4pt)

(c) Under what conditions will the echo be perfectly cancelled? (3pt)

(d) Assume both the local caller and the remote caller are listening to the same radio channel during the phone call with the adaptive filter in operation. Describe what will happen. (5pt)

Solution:

(a) Tracing the remote caller signal we first go through G then arrive at the local site. At the local site the sound is transmitted by the loudspeaker through the room to the microphone (modeled as H) and then back again to the remote site through channel G again. The signal from the local caller is picked up by the microphone and sent through the channel G to the remote site. The following left block diagram below detail the setup.



(b) To mitigate the echo effect we add a filter $\hat{H}$ at the local site as illustrated to the right above. It is feed with the signal from the remote site and the updating is based on minimizing the variance of the error signal $e(n)$ (the microphone signal subtracted with the output of the adaptive filter) for example using the LMS algorithm.

(c) When the filter $\hat{H}$ is equal to the acoustic channel H the echo is cancelled perfectly. If the length of impulse response of the acoustic channel is shorter then the number of filter coefficients in the adaptive FIR filter this is possible to achieve. If the local caller is silent the convergence will be exactly to the optimal filter. However, if the local caller also speaks the filter will move around the optimal point but stay close if we assume the remote and the local callers are uncorrelated (which is the normal case).

(d) When we have the same radio source at the local site as well as at the remote site we have an additional signal component present, $r(t)$, the audio signal from the radio. The error will then be

$$e = s_l + r + (h - \hat{h}) * g * (s_r + r) = s_l + (1 + (h - \hat{h}) * g) * r + (h - \hat{h}) * g * s_r$$

A natural assumption here is that the signals s_l, s_r and r are all uncorrelated. Minimizing the variance of the error w.r.t. $\hat{h}$ yields

$$\begin{aligned}\hat{\mathbf{h}}_{\text{opt}} &= \arg\min_{\hat{\mathbf{h}}} \mathbf{E}\, e(n)^2 = \\ &= \arg\min_{\hat{\mathbf{h}}} \frac{1}{2\pi} \int_0^{2\pi} |1 + (H - \hat{H})G|^2 S_{rr} + |H - \hat{H}|^2 |G|^2 S_{s_r s_r}\, d\omega\end{aligned}$$

Clearly in this case the minimum yields (in general) $\hat{H} \neq H$ and the echo canceller will not remove the echo as desired.

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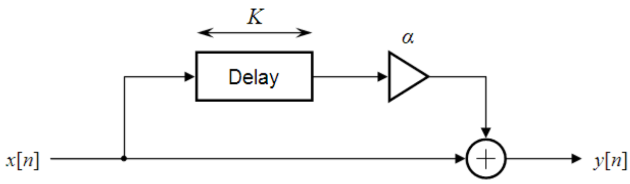
4. The comb filter is a filter with the general structure as illustrated below



which means that the filtered output is the present input added with the input delayed K samples, where K is a positive integer, and scaled with α.

(a) Explain why the filter is particularly inexpensive to implement if $\alpha = \pm 1$ (2pt)

(b) Derive the frequency function for the comb filter (3pt)

(c) Which choice $\alpha = +1$ or $\alpha = -1$ would yield a filter where the lowest frequencies will pass through the filter? (2pt)

(d) Assume $\alpha = -1$. At which frequencies will the frequency function have magnitude zero? *Hint:* Recall the complex solutions to the algebraic equation $x^K = 1$ (3pt)

Solution:

(a) In this case the filter does not need any multiplier which hence saves speed or circuit space depending on the implementation platform.

(b) $y(n) = x(n) + \alpha x(n-K)$ which leads to the frequency function $H(\omega) = 1 + \alpha e^{-j\omega K}$

(c) For $\alpha = 1$ we obtain $H(0) = 2$ and for $\alpha = -1$ we obtain $H(0) = 0$. Hence, $\alpha = 1$ is the correct choice.

(d) solutions to $H(\omega) = 0$ then translates to solutions to $1 = e^{j\omega K}$. All solutions to this equation are $\omega = \frac{2\pi}{K} l$ where l is an arbitrary integer.

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5. The DFT and IDFT pairs are defined as:

$$\text{DFT:} \quad X(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi kn/N}, \quad \text{IDFT:} \quad x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) e^{j2\pi kn/N}$$

The Fast Fourier Transform (FFT) is a computational method to efficiently calculate the DFT. Assume the FFT implementation of DFT is available as a function FFT in some computer language (or hardware). Show how you can use the FFT function to also calculate the IDFT by introducing some elementary operations on the input/output data. *Hint:* Use conjugation! (6pt)

Solution: If we employ complex conjugation to the IDFT equation we obtain

$$x^*(n) = \left(\frac{1}{N} \sum_{k=0}^{N-1} X(k) e^{j2\pi kn/N} \right)^* = \frac{1}{N} \sum_{k=0}^{N-1} X^*(k) e^{-j2\pi kn/N}$$

Since the last summation is the DFT of X^* we can calculate the IDFT in the following way. 1) Complex conjugate X, i.e. change sign of all the imaginary parts. 2) use the DFT function 3) complex conjugate the result. 3) Divide all values with N. ■

END - Good Luck!