Examination
SSY130 Applied Signal Processing
Suggested solutions

14:00-18:00, December 14, 2010

Instructions

- *Responsible teacher:* Tomas McKelvey, ph 8061. Teacher will visit the site of examination at 14:45 and 16:00.
- Score from the written examination will together with course score determine the final grade according to the Course PM.
- Your preliminary grade is reported to you via email.
- Exam grading review will be held in the "Blue Room" on level 6 at 12:00-13:00 on January 19, 2010.

Allowed aids at exam:

- L. Råde and B. Westergren, Mathematics Handbook (any edition, including the old editions called Beta).
- Any calculator
- One a4 size single page with written notes

Other important issues:

- The ordering of the questions is arbitrary.
- All solutions should be well motivated and clearly presented in order to render a full score. Unclear presentation or adding, for the problem in questing, irrelevant information render a reduction of the score.
- Write solutions to each problem on a separate sheet of paper.
- The maximum score is 52 points.

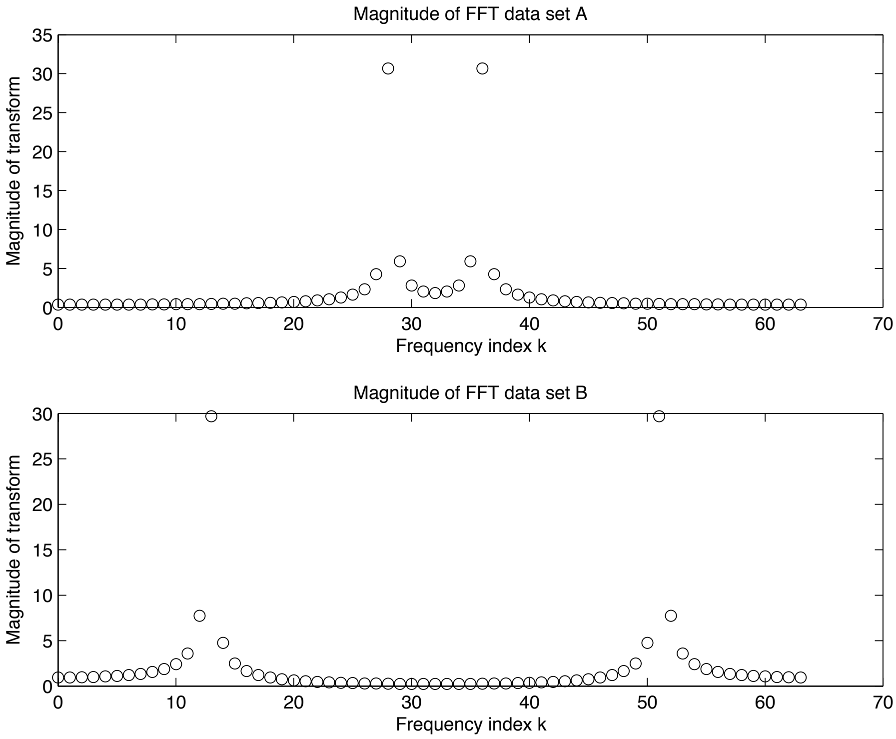

Figure 1: 64 point FFT for problem 1

Problems

1. A continuous time real valued signal containing one sinusoidal component is sampled without any antialiasing filter and 64 samples are collected.

   (a) Assume the frequency of the sinusoidal component is less than 12.5 kHz and that the signal is sampled with a sample rate of 25 kHz. The magnitude of the 64 point FFT of the signal is illustrated in the top graph in Figure 1. The first peak appear at index 28. Derive the index of the second peak and determine the frequency (in Hz) of the sinusoidal component. (3pt)

   (b) Assume we know that the frequency of the sinusoidal component is larger than 62.5 kHz and less than 70 kHz and the sample rate is still 25 kHz. The resulting FFT is again illustrated in the top graph in Figure 1. What is the frequency of the sinusoidal component? (4pt).

   (c) Assume we know that the frequency is less than 100 kHz and the signal is sampled twice. The first 64 samples (data set A) are collected with a sample rate of 25 kHz and the second batch of 64 samples (data set B) are collected with 30 kHz sample rate. Top graph in Figure 1 corresponds to the sample rate of 25 kHz and the bottom graph corresponds to the sample rate of 30 kHz. In the bottom graph the first peak is located at index 13. What is the frequency of the sinusoidal signal? (5pt)

   Solution:

   (a) Since the signal is real the amplitude curve is symmetric around the Nyquist frequency. Hence the index of the second peak is $N - \text{index1} = 64 - 28 = 36$. The frequency of the signal is $f_0 = \text{index1} * f_s/N = 28 * 25/64 = 11kHz$ (rounded to zero decimals).

   (b) Since the signal frequency is higher than the Nyquist frequency, aliasing will occur. Interval $[62.5, 75]$ kHz will in the sampled signal appear in the interval $[12.5, 25]$ kHz with a mirror in frequency interval $[0, 12.5]$ kHz. If f_0 is the frequency of the continuous signal then the FFT will have a peak between 12.5 kHz and $f_s = 25$ kHz at

$$f_1 = f_0 - kf_s = 36 * 25/64 = 14\,[\text{kHz}],$$

where k is an integer. With $k = 2$ and $f_0 = 64$ kHz the equation is satisfied and we obtain $f_0 = 64$ kHz.

(c) Using the relationship above possible solutions,for the 25 kHz sample rate are $11 + k * 25$ which gives potential solutions

$$\{-89, -64, -39, -14, 11, 36, 61, 86\}$$

the second peak gives solutions $14 + k * 25$ which correspond to potential solutions

$$\{-86, -61, -36, -11, 14, 39, 64, 89\}.$$

In summary, since the signal is real the frequency must be in the set

$$\mathcal{S}_1 = \{11, 14, 36, 39, 61, 64, 86, 89\}.$$

For the second data set sampled at 30 kHz the peaks are located at index 13 and index 64-13 = 51. Potential frequencies of the original signal are thus $13 * 30/64 + k * 30$ or $51 * 30/64 + k * 30$ which yields the set

$$\mathcal{S}_2 = \{6, 24, 36, 54, 66, 84, 96\}.$$

The only common element in $\mathcal{S}_1$ and $\mathcal{S}_2$ is 36 and hence the frequency is 36 kHz.

□

2. This problem is about signal interpolation and it's implementation.

(a) Describe the two processing blocks usually found in a linear signal interpolation function. (2pt)

(b) Assume the orignal real signal has a sample rate of 2 kHz. Describe an interpolation design which will change the sample rate to 10 kHz but retain all the original signal bandwidth. Specify the edge frequencies of the passband of the filter involved. (3pt)

(c) What is a polyphase filter representation? (2pt)

(d) Write a Matlab code which implements the two steps in an interpolator which increases the sample rate with a factor of 3. You can assume that the FIR fiter involved has a length which is an integer multiple of 3. Write the code as a function where the filter coefficients and the input signal are input vectors and the output of the function is a vector with the interpolated output signal. For a full score your implementation should use as few computational resources as possible. (5pt)

Solution:

(a) A signal interpolater which increases the sample rate a factor L can be constructed by up-sampling the signal, (i.e. inserting $L-1$ zero samples between the original samples) and as a second step apply a low pass filter to remove the $L-1$ images in the spectrum caused by the up-sampling operation.

(b) To change from 2 to 10 kHz we need a factor five ($L = 5$) upsampling operation. The final low-pass filter should retain the original signal information and hence should have a cut off frequency at 1 kHz which is the Nyquist frequency of the original signal.

(c) A polyphase filter representation is the decomposition of an orignal FIR filter $H(z)$ into a sum of polyphase components A_i such that

$$H(z) = \sum_{k=0}^{M-1} h(k)z^{-k} = \sum_{i=0}^{L-1} z^{-i}A_i(z^L)$$

where

$$A_i(z) = \sum_{k=0}^{P-1} a_k z^{-k}$$

and $P = \text{ceil}(M/L)$. With an interpolation filter in polyphase form usage of the noble identity the up-sampler and the filtering blocks can be interchanged and the filtering can be performed at the low rate side.

(d) First, considering the filtering step we have that the output y at time index $mL + l$ is given by

$$y(mL + l) = \sum_{k=0}^{M-1} h(k)x_u(mL + l - k)$$

where x_u are the upsampled signal with $L - 1$ zeros between the nonzero values. Since x_u is non-zero only for indices which is a factor of L we can rewrite the summation. Set $P = M/L$. x_u is nonzero only for indices satisfying $l - k = -pL$ for integers p. Hence we obtain

$$y(mL + l) = \sum_{p=0}^{P-1} h(pL + l)x_u(mL - pL) = \sum_{p=0}^{P-1} h(pL + l)x(m - p)$$

where $l = 0, 1, \ldots, L - 1$ and m any integer. This is for each l a convolution involving only a filter of length $P = M/L$. A matlab function would then be

```
function y = interpolation(x,h,L);
h = h(:);
x = x(:);
M = length(h);
P = M/L;
Nx = length(x);
Nysub = Nx+P-1;
idxy = 0:Nysub-1;
idxh = 0:P-1;
y = zeros(Nysub*L,1);

for l=1:L,
  y(idxy*L+l) = conv(h(idxh*L+l),x);
end
```

Of course in some caseses the convolution in the for-loop could be exchanged by filtering using FFT to increase the efficency even further.

```
function y = interpolation_fft(x,h,L);
h = h(:);
x = x(:);
M = length(h);
P = M/L;
Nx = length(x);
Nysub = Nx+P-1;
idxy = 0:Nysub-1;
```

```
idxh = 0:P-1;
Nfft = pow2(ceil(log2(P+Nx-1)));
y = zeros(Nysub*L,1);
X = fft(x,Nfft);
for l=1:L,
  tmp = ifft(X.*fft(h(idxh*L+l),Nfft));
  y(idxy*L+l) = real(tmp(1:P+Nx-1));;
end
```

□

3. A loudspeaker telephone or a mobile phone with handsfree need a signal processing functionality known as an echo canceler. When the audio from the remote party is sent through the loudspeaker of the local unit it is propagated through the room and then picked up by the microphone of the same unit. If no echo cancelation is functional, the remote party will hear his/her own audio again with a delay and hence, is called an echo. Discuss a signal processing solution for removal of the echo and motivate your system solution based on properties of the application. Keep your discussion at a systems/algorithm level and provide a diagram showing all involved signals, filters and other processing units. (10pt)

   Solution: The system need to remove the audio contribution from the remote party which is sensed by the microphone, we denote this signal $x(t)$. The audio from the remote party $y(t)$ is sent through the loudspeaker and into the room. The sensed signal $x(t)$ is thus a filtered version of the $y(t)$ signal and the filter $H(z)$ corresponds mainly to the acoustic effect of the room. Since the loudspeaker system need to function in many environment an adaptive filtering solution is suitable which take into account both changing room acoustics as well as the spectrum of different speakers. A schematic view of the system can be seen in the textbook Figure 8.14 (a). □

4. This problem is about an LMS filtering application where the filter operates in a system modeling fashion. Below is a number of statements about the behavior of the filter. Give a brief explanation of a possible cause of the described behavior.

   (a) First the filter seems to work fine but suddenly when the volume of the input increased the filter values became extremely large and/or out of the numerical range of the computer. (2pt)

   (b) If the convergence time is short more background noise is heard and vice versa. (2pt)

   (c) First a white noise input is used in order to train the filter. Then the updating is turned off and the filter is used as a fixed filter for input signals of varying character. However, the fixed filter gives a higher error variance as compared to when the updating was enabled all the time. (2pt)

   (d) The amplitude of the error converges quickly to a low value but the fiter coefficients does not match the ones for the modeled system. (2pt)

   Solution:

   (a) Due to the volume (amplitude) change of the input signal the adaptive filter will take larger steps in the parameter space. Here, the filter coefficients seems to diverge which indicates that the step length has become too large.

   (b) The residual error is the error signal which is created by the movement of the filter coefficients around the optimal point. The variance of this error increases with the size of the step length coefficient μ. A large μ also leads to short convergence time. Hence there is a trade-off between short convergence time and small residual error variance.

(c) If we assume the complexity of the system we model is more complex than the FIR filter in the LMS algorithm, white noise training signal will lead to a filter which optimizes

$$\int_{-\pi}^{\pi} |H(\omega) - \hat{H}(\omega)|^2 \, d\omega$$

which will give an approximation which gives the minimum average mean-squared error. However, if the input signal is narrow band then the mean square error could be smaller since an active LMS filter then only will adapt to the limited band where the input signal has power and hence can make the mean square error in this band smaller.

(d) Here it appears as if the input is narrow band such that the autocorrelation matrix Φ_{yy} is singular, hence the optimum to the Wiener filter problem

$$\Phi_{yy}\mathbf{h} = \Phi_{yx}$$

has an infinite number of solutions (including the one which models the system). An LMS algorithm will converge to only one of them and most likely not to the point corresponding to the full system.

□

5. A discrete time causal linear system has a periodic input with a period of P samples. Assume one period of the output is known. Describe a computational procedure which calculates the input to the system for the following two cases:

   (a) The linear system is FIR and has a known impulse response $h(n)$ with a length less than P. (4pt)

   (b) The linear system is FIR and has a known impulse response $h(n)$ with a length larger than than $P + 1$. (6pt)

   Solution: Let $Y(k)$ and $X(k)$ denote the length P DFT of the output and input respectively. Then since the DFT of one period of a periodic signal equals the Fourier series coefficients we directly have $Y(k) = H(\omega_k)X(k)$ where $\omega_k = 2\pi k/P$.

   (a) Since length of the impulse response is less then P we have $H(\omega_k) = H(k)$ where $H(k)$ is the length P DFT of the impulse response of the system zero padded to a total length of P. Consequently we can calculate the input as $x(n) = \text{IDFT}[Y(k)/H(k)]$

   (b) For this case the impulse response length is longer than the period time and we can thus not directly use the length P DFT to get $H(\omega_k)$. Let N_h denote the length of the impulse response. We can use the definition of the DTFT to calculate $H(\omega_k) = \sum_{n=0}^{N_h-1} h(n)e^{-j2\pi nk/P}$ and then calculate the input as in (a) □

END

Good Luck!