Name:

Exam in the course Antenna Engineering 2016-06-03

ANTENNA ENGINEERING (SSY100)
(E4) 2015/16 (Period IV)

Friday 3 June 1400-1800 hours.

Teachers: Associate Prof. Jian Yang, Associate Prof. Marianna Ivashina, Associate Prof. Rob Maaskant, Assistant Prof. Ashraf Uz Zaman.

Questions: Jian Yang, Tel. 1736, Mobil: 0733 678841

The exam consists of 2 parts. Part A is printed on colored paper and must be solved without using the textbook. When you have delivered the colored text and the solutions of Part A (latest 17:00), the textbook can be used for Part B of the exam.

You are allowed to use the following:

For Part A:	**Pocket calculator of your own choice**
For Part B only:	**Mathematical tables including Beta**
	Pocket calculator of your own choice
	Kildal's compendium "Foundations of Antennas: A Unified Approach for LOS and Multipath"

(The textbook can contain own notes and marks on its original printed pages. No other notes are allowed.)

Tentamen består av 2 delar. Del A har tryckts på färgade papper och skall lösas utan att använda läroboken. När du har inlämnat dom färgade arken med uppgifterna för del A och dina svar på dessa uppgifter (senast 17:00), kan du ta fram läroboken för att lösa del B.

Tillåtna hjälpmedel:

För del A:	Valfri räknedosa
För del B:	Matematiska Tabeller inkluderad Beta
	Valfri räknedosa
	Kildals lärobok "Foundations of Antennas: A Unified Approach for LOS and Multipath"

(Boken kan innehålla egna noteringar skrivna på de inbundna sidorna. Extra ark med noteringar tillåts inte.)

Name:

Name:

PART A (must be delivered before textbook can be used)

1.0 Foundations of Antenna Engineering (25p)

1.1. Write out the general formula of the far field function of a y-polarized BOR1 antenna. Write out the co-polar and cross-polar vectors of the Ludwig's third definition for this antenna. Then, write the co-polar and cross-polar far-field function of the y-polarized BOR1 antenna. (6p)

Answer:

The far field function of a y-polarized BOR1 antenna:

$$\boldsymbol{G}_y(\theta,\varphi) = G_E(\theta)sin\varphi\hat{\theta} + G_H(\theta)cos\varphi\hat{\boldsymbol{\varphi}}$$

The far field function of an x-polarized BOR1 antenna (for cross-polar of Ludwig 3rd definition):

$$\boldsymbol{G}_x(\theta,\varphi) = G_E(\theta)cos\varphi\hat{\theta} - G_H(\theta)sin\varphi\hat{\boldsymbol{\varphi}}$$

The Ludwig third definition is the special case of BOR1 when

$$G_E(\theta) = G_H(\theta) = 1$$

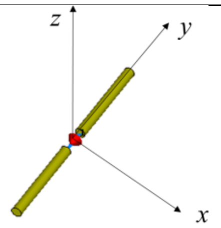
So

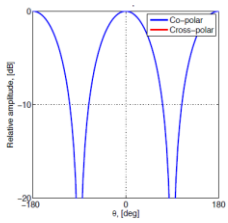
$$\widehat{\boldsymbol{co}} = sin\varphi\hat{\theta} + cos\varphi\hat{\boldsymbol{\varphi}}$$
$$\widehat{\boldsymbol{xp}} = cos\varphi\hat{\theta} - sin\varphi\hat{\boldsymbol{\varphi}}$$

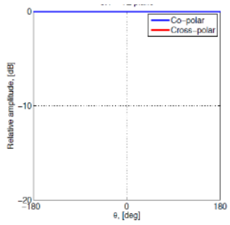
So

$$G_{yco}(\theta,\varphi) = \boldsymbol{G}_y(\theta,\varphi)\cdot\widehat{\boldsymbol{co}}^* = G_E(\theta)sin^2\varphi + G_H(\theta)cos^2\varphi$$
$$G_{yxp}(\theta,\varphi) = \boldsymbol{G}_y(\theta,\varphi)\cdot\widehat{\boldsymbol{xp}}^* = G_E(\theta)sin\varphi cos\varphi - G_H(\theta)sin\varphi cos\varphi$$

1.2. State which plane is the E-plane and which plane is the H-plane for the following antennas, and sketch the radiation patterns in E- and H-planes. (10p)

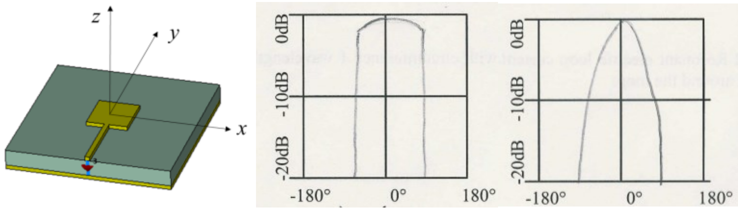
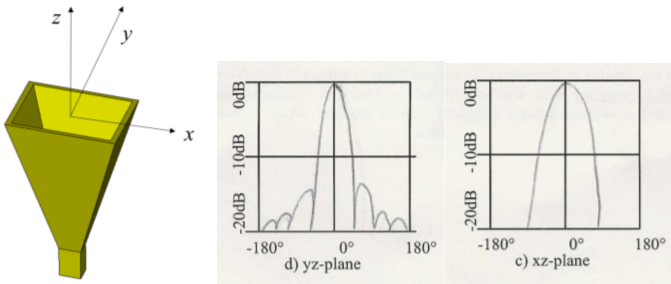
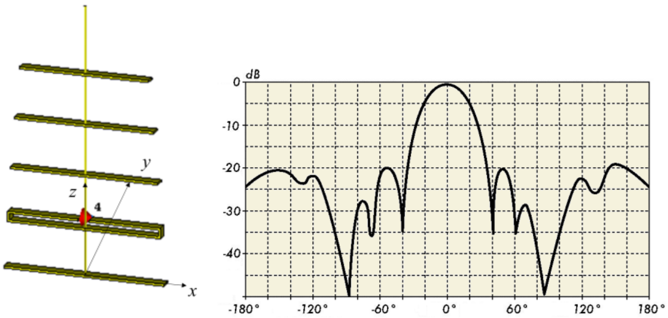
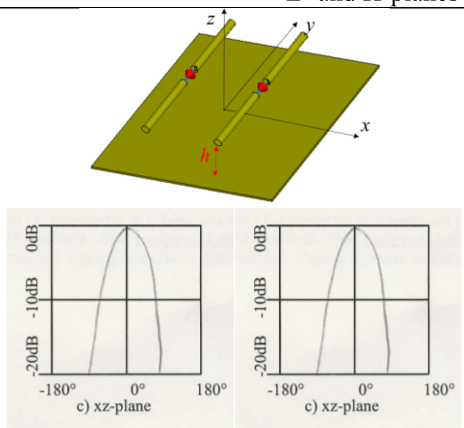
<table>
<tr>
<td>(a) A half-wave dipole along y axis
Answer
E-plane: YZ plane
H-plane: XZ plane</td>
<td>



E-plane


H-plan</td>
</tr>
</table>

Name:

(b) A microstrip patch antenna ***Answer*** E-plane: YZ plane H-plane: XZ plane	 E-plan H-plan
(c) A rectangular pyramidal horn ***Answer*** E-plane: YZ plane H-plane: XZ plane	 E-plan H-plan
(d) Yagi antenna ***Answer*** E-plane: YZ plane H-plane: XZ plane	 E- and H-planes
(e) Eleven antenna: two half-wave dipoles located above a ground plane with a height of h and with a spacing of half wavelength in between. ***Answer*** E-plane: YZ plane H-plane: XZ plane	 E-plane H-plane

1.3. Write out the directivity, radiation resistance and radiation reactance for the following antennas: (i) short dipole; (ii) halfwave dipole; and (iii) monopole over a ground plane. You can use the words “very small” or “very large”. (3p)

Answer:

antennas	Directivity (dBi)	Radiation resistance (Ohm)	Radiation reactance (Ohm)
Short dipole	1.73	Very small	Negative very large
Half-wave dipole	2.16	73	0
Monopole over a ground plane	5.16	36.5	0

1.4. What is the difference between the antenna gain and the antenna directivity? Write out the expression of the relation between the antenna gain and the antenna directivity. Explain the terms you use. (2p)

Answer:

$$G_0 = e_{rad} e_{pol} D_0$$

G_0 is the gain, D_0 is the directivity, e_{rad} is the radiation efficiency including mismatch and ohmic loss in antenna, e_{pol} is the polarization efficiency of alignment.

1.5. Please answer the following questions. (2p)

a) What is the level (relative to the main beam) of the first sidelobe of a large rectangular uniform plane PEC aperture?

b) What is the level (relative to the main beam) of the first sidelobe of a large rectangular uniform plane free space aperture?

c) What is the level (relative to the main beam) of the first sidelobe of a large circular uniform plane free space aperture;

d) A large circular uniform plane free space aperture has an area of A, what is the directivity of the aperture?

Answer:

a)-13.2 dB; b)-13.2 dB; c)-17.6 dB; d) $D_0 = \frac{4\pi}{\lambda^2} A$.

1.6. A large linear array has 40 elements with the element spacing of 0.8λ and uniform excitation, as shown in Fig. 1.1. From array theory, we know that the main beam direction is normal to the array line and there are no grating lobes. But by superposition, this array can be built up by two sub-arrays with 20 elements and element spacing of 1.6λ, with a shift of 0.8λ between the two sub-arrays, as shown in Fig. 1.1. By array theory, both sub-arrays have grating lobes. It seems that there is a conflict between the two methods: direct array method and two-sub-array method. State if the two-sub-array method is correct or not, and give your arguments. (2p)

Answer:

The two-sub-array method is correct. Both sub-arrays have grating lobes but they are in the same direction and out of phase (180 degree phase difference) so they are canceled each other. Actually, the grating lobe appears when the phase

difference of neighboring elements in a direction is 360 or its times (main beam has zero phase difference of neighboring elements). So if the sub arrays have one direction that the neighboring elements have 360 degree difference, the two sub-arrays have 180 degree difference so the grating lobes cancel each other.
We will use this two-sub-array method in Question 4.

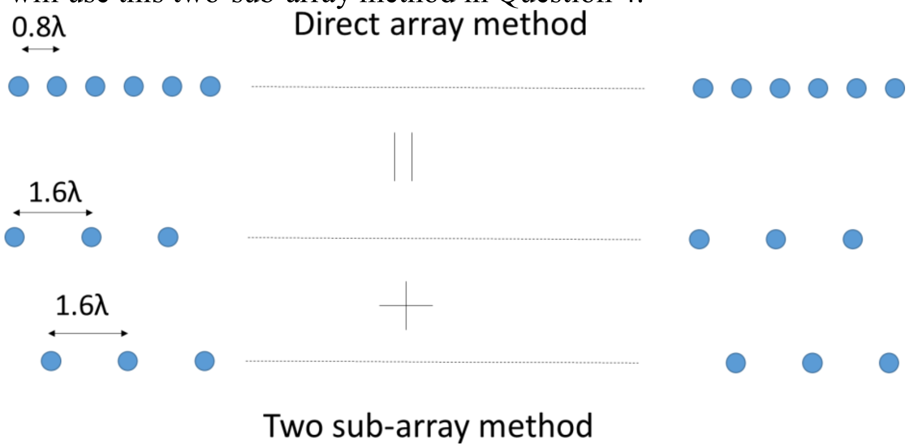


Fig. 1.1 Linear array analysed with direct array method and two-sub-array method

2.0 Antenna Systems in Multipath Environment (25p)

2.1 Briefly explain the concept of multipath fading, and the mechanisms behind fast fading and shadowing. (2p)

Answer:

Multipath fading is the received signal <u>variation</u> when the receiver moving around in a multipath environment. The waves arrive from multiple paths due to scattering and impinge at the receiver inducing voltages that interfere with each other constructively and destructively.

2.2 Briefly explain what the cumulative distribution function (CDF) of the received signal represents. (2p)

Answer:

CDF shows the cumulative probability of received signal amplitude in dB in multipath fading environment.

2.3 What is the definition of a Rich Isotropic MultiPath (RIMP) environment? (2p)

Answer:

RIMP is defined as: 1)<u>Isotropic</u>. This means that the Angle-of-Arrivals are uniformly distributed over a sphere of unit radius, i.e., the likelihood of observing a wave impinging at the receiver is the same for all directions. 2)<u>Power balanced</u>. This means that the cross-polar power discrimination equals 1 in linear scale or 0 dB in dB-scale, i.e., the average received power of a vertical polarization is equal to the power in the horizontal polarization.

2.4 Briefly explain the reasons that a reverberation chamber can emulate the RIMP environment. (2p)

Answer:

Reverberation Chamber is a big cavity so there are many modes in it. Each mode consists 8 incident waves. Study shows that the AOAs of the incident waves are uniformly distributed over a sphere of unit radius. Using 3 wall antennas we can have a power balanced environment. So RIMP is emulated by RC.

2.5 What is the fundamental parameter that characterizes the performance of a single-port antenna in rich isotropic multipath (RIMP)? (2p)

Answer:

The total radiation efficiency.

2.6 What is the difference between the total radiation efficiency of a lossless single-port antenna and the total embedded efficiency of a given port in a lossless multi-port antenna in free-space? (2p)

Answer:

The total radiation efficiency of a lossless single-port antenna in free-space is determined by the matching efficiency $e_{totrad} = 1 - |S_{11}|^2$, while in the multi-port case the total embedded efficiency is given by the decoupling efficiency

$$e_{totemb,i} = 1 - \sum_j |S_{ij}|^2$$

2.7 Briefly explain the main idea behind diversity and mention three antenna diversity methods. (4p)

Answer:

The idea is to exploit received signals with low correlation at different antennas. Fading dips do not occur simultaneously at the different antennas in multipath environments, especially in rich isotropic multipath channels.
Polarization diversity, space diversity, pattern diversity.

2.8 What is the main differences between an anechoic chamber and a reverberation chamber? Which propagation environments do these two chambers represent respectively? (3p)

Answer:

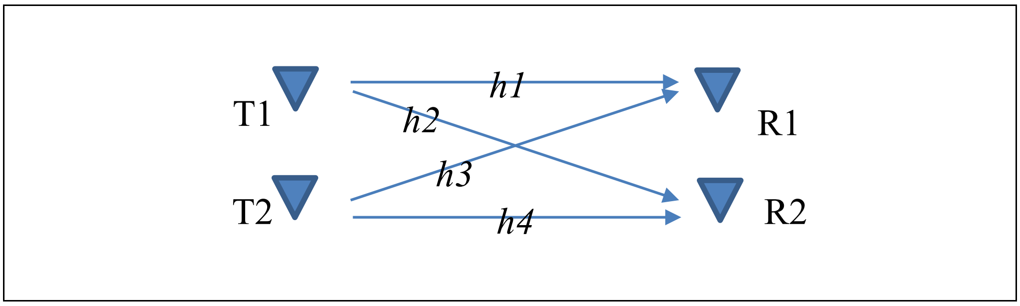
In an anechoic chamber (AC) absorbers are placed on the walls, floor and ceiling of the chamber to attenuate all the waves except the direct line-of-sight wave between the antenna under test and the chamber antenna. In reverberation chambers (RC) highly reflective walls and low absorption are used to create a cavity resonator with high Q-factor. The resulting standing wave pattern is stirred by moving reflectors to create a Rayleigh fading environment.
AC: line of sight (free space)
RC: RIMP

2.9 Explain what apparent, effective and actual diversity gains are. You can use sketch to explain (3p)

Answer:

Apparent diversity gain: The reference CDF is the CDF at the port with the <u>strongest average power levels</u>.
Effective diversity gain: The reference CDF is the CDF at the port of the ideal reference antenna, i.e., <u>the theoretical Rayleigh</u> CDF.
Actual diversity gain: The reference CDF is the CDF at the port of an <u>existing practical single-port</u> antenna. The latter is then normally the single-port antenna that is to be replaced by the diversity antenna under test, so that we can determine the actual improvement over an existing solution.

2.10 You have a 2x2 MIMO system, and you know the channels, as shown in the figure below. How do you combine the signals by using MRC algorithm? (3)



Answer:

$$C_{MRC}(t) = \sum_{i=1}^{4} C(t)h_i \cdot h_i^*$$

2.11 Write the expression for the throughput in conducted measurements based on the threshold receiver model. Explain the terms you use. (2p)
Answer:

A: $TPUT_{cond}\left(P\right) = TPUT_{\max} \begin{cases} 0 & if \quad P < P_{th} \\ 1 & if \quad P \geq P_{th} \end{cases}$

PART B (You can use the textbook to solve these problems, but only after your solutions to PART A has been handed in)

3.0 Boundary Conditions, Reaction Concept, Equivalent Circuit of Antennas (25p)

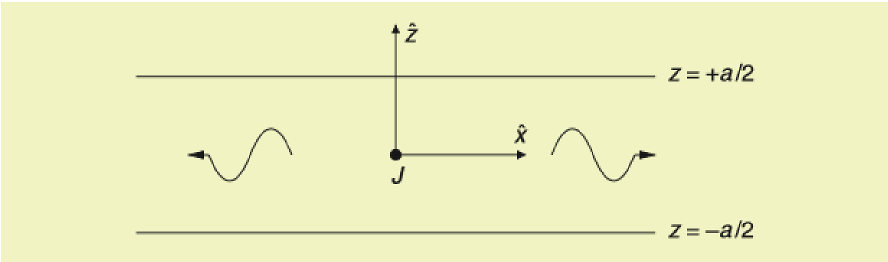


Fig 3.0 A parallel-plate waveguide excited by a line source.

On-chip antennas for chip-to-chip RF data transmission is a hot antenna research topic. The simplest model for such antenna radiation can be described by Fig. 3.0, where electromagnetic waves excited by antennas propagate between a parallel-plate waveguide. The two PEC ground planes are at $z = \pm a/2$. The expression for the time-harmonic electric field $\boldsymbol{E}(x, z, \omega)$ inside the waveguide that is radiated by the electric line current $\boldsymbol{J} = I_0\delta(x)\delta(z)\,\hat{\boldsymbol{y}}$ is given by

$$E_y(x, z) = \omega\mu I_0 \sum_{n=0}^{\infty} \frac{(-1)^n \sin(k_{z,n}(|z| - a/2))}{k_{x,n}a} e^{-jk_{x,n}|x|} \tag{3.1}$$

where $k_{z,n} = (\pi/a)(1 + 2n)$; $k_{x,n} = \sqrt{k^2 - k_{z,n}^2}$, and $k = 2\pi/\lambda$.

3.1 Show that the above E-field satisfies the boundary conditions at the PEC ground planes, and explain which term in (3.1) describes that the field propagates away from the source. Assume that $k_{x,n} \neq 0$. (3p)

Answer:

The tangential component of the E-field must vanish at $z = \pm a/2$, that is $E_y(x, \pm a/2) = 0$, which is seen to occur for any term n in the sum, since $\sin(0) = 0$, while $k_{z,n}$ and $k_{x,n}$ are both nonzero and finite. The field also satisfies the radiation condition because the term $e^{-jk_{x,n}|x|}$ represent outward traveling waves from the line source, for each n

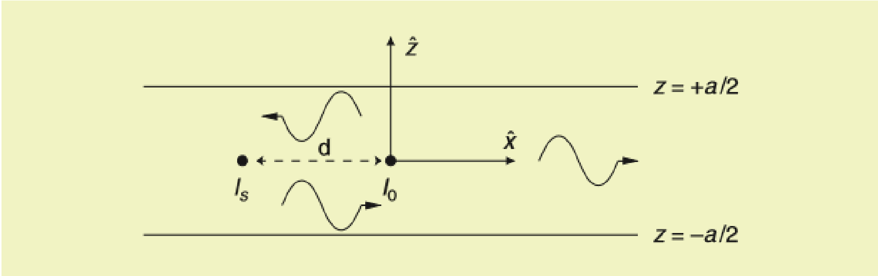


Fig 3.1 A PEC reflecting wire at a distance d from the "driving wire" to realize unidirectional wave propagation.

Next, we place a PEC wire at $x = -d$. That is, a distance d away from the line source to reflect the E-field from the line source for realizing unidirectional wave propagation.

3.2 To find the E-field inside the waveguide in the presence of the PEC wire, we remove the PEC wire and replace it by a line source $\boldsymbol{J}_s = I_s\delta(x+d)\delta(z)\,\hat{\boldsymbol{y}}$ with (unknown) amplitude I_s. This line source will represent the physically induced wire current. What equivalence principle is used, and why do we want to make use of this specific type of equivalence principle? (2p)

Answer:

The Physical Equivalence Principle, since the PEC wire has been replaced by the physically induced current that has now become a source current radiating in the waveguide region. We make use of equivalence principles to simplify the problem. In this specific case the E-field becomes the sum of the E-fields from both line currents, the individual expressions of which are known, which is simpler than to find the E-field from a source in the presence of a PEC wire.

3.3 After invoking the above equivalence principle, we now have the two radiating source currents $\boldsymbol{J}_0 = I_0\delta(x)\delta(z)\,\hat{\boldsymbol{y}}$ and $\boldsymbol{J}_s = I_s\delta(x+d)\delta(z)\,\hat{\boldsymbol{y}}$. Derive an expression for the total E-field inside the waveguide through the superposition principle. Accordingly, impose the correct boundary condition for the so-obtained total E-field at the field reflecting PEC wire and show that the expression for the unknown amplitude I_s is given by: $I_s = -I_0\left(\sum_{n=0}^{\infty}\frac{e^{-jk_{x,n}d}}{k_{x,n}}\right)/\left(\sum_{n=0}^{\infty}\frac{1}{k_{x,n}}\right)$ (5p).

Answer:

With the expression for E_y given in (3.1), the total E-field inside the waveguide simply becomes the field from two line sources, i.e.,

$$E_y^t(x,z) = E_y(x,z) + \frac{I_s}{I_0}E_y(x+d,z)$$

The tangential component of the total E-field must vanish at the PEC wire, that is, $E_y^t(x=-d, z=0) = 0$, this leads to

$$E_y(x=-d, z=0) + \frac{I_s}{I_0}E_y(x=0, z=0) = 0$$

so that the solution for I_s is given by

Name:

$$I_s = -I_0 \frac{E_y(x=-d, z=0)}{E_y(x=0, z=0)} = -I_0 \left(\sum_{n=0}^{\infty} \frac{e^{-jk_{x,n}d}}{k_{x,n}} \right) / \left(\sum_{n=0}^{\infty} \frac{1}{k_{x,n}} \right)$$

3.4 Next, we consider the line current $\boldsymbol{J}_0 = I_0 \delta(x)\delta(z)\,\hat{\boldsymbol{y}}$ and the incremental dipole source $\boldsymbol{J}_s = I_s \ell_s \delta(x+d)\delta(y)\delta(z)\ \hat{\boldsymbol{y}}$ inside the waveguide. Give an expression for the mutual reaction $\langle \boldsymbol{E}_s, \boldsymbol{J}_0 \rangle$, where $\boldsymbol{E}_s$ is the field inside the waveguide generated by $\boldsymbol{J}_s$. Hint: make use of the reciprocity theorem for the symbolic evaluation of the integral (3p).

Answer:

The reaction integral is given by:

$$\langle \boldsymbol{E}_s, \boldsymbol{J}_0 \rangle = \iiint_{-\infty}^{+\infty} \boldsymbol{E}_s \cdot \boldsymbol{J}_0 dV = I_0 \int_{-\infty}^{+\infty} E_{y,s}\,(x=0, y, z=0) dy$$

However, this integral is difficult to evaluate due to the oscillating behavior of $E_{y,s}$ as a function of y. Instead, we can make use of reciprocity and directly evaluate it as:

$$\langle \boldsymbol{E}_s, \boldsymbol{J}_0 \rangle = \langle \boldsymbol{E}_0, \boldsymbol{J}_s \rangle = \iiint_{-\infty}^{+\infty} \boldsymbol{E}_0 \cdot \boldsymbol{J}_s dV = I_s \ell_s E_y(x=-d, z=0)$$

where E_y is already given through Eq. (3.1).

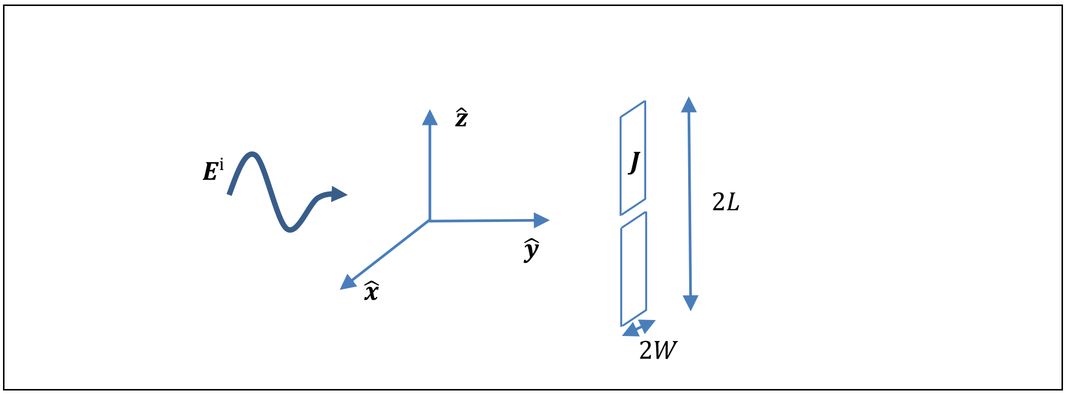


Fig 3.1 A strip dipole excited by an incident plane wave.

3.5 Consider Fig. 3.1, which shows a $\lambda/2$ strip dipole antenna placed at $y = L$. The surface current distribution, when excited by a current source $I = 1$ A across the infinitesimal strip gap at $z = 0$, is $\boldsymbol{J} = \frac{1}{2W} \cos\left(\frac{\pi z}{2L}\right) \hat{\boldsymbol{z}}$ A/m with finite support $-L \leq z \leq L$ and $-W \leq x \leq W$. The antenna self-admittance is 0.01 Siemens. Given these transmitting characteristics, consider now the receiving situation, where this antenna is excited by the plane wave field $\boldsymbol{E}^{\mathrm{i}} = e^{-jky}\hat{\boldsymbol{z}} - e^{-jk(y+\lambda/4)}\hat{\boldsymbol{x}}$, where k is the free-space wavenumber. Assume the time dependence $e^{j\omega t}$.

(a) Is the incident field left- or right-hand, circularly, or elliptically polarized? (2p)

Answer:

Take $y = 0$, as an example, and we observe that

$$\boldsymbol{E}^{\mathrm{i}}(y = 0, t) = \mathrm{Re}\big(e^{j\omega t}\boldsymbol{E}^{\mathrm{i}}(y = 0)\big) = \cos(\omega t)\,\hat{\boldsymbol{z}} - \sin(\omega t)\,\hat{\boldsymbol{x}}$$

which is a vector that rotates counter clockwise with time and with respect to the propagation direction $\hat{\boldsymbol{y}}$, thus the field is left-circularly polarized.

(b) Compute the complex-valued output voltage across a 50 Ohm antenna port termination for the case that $L = \lambda/4$. Hint: first find the antenna open-circuit voltage, then construct the equivalent circuit of a receiving antenna and then compute the output voltage across the load. (5p)

Answer:

Using the reaction integral formula $V = -\frac{1}{I}\langle \boldsymbol{E}^{\mathrm{i}}, \boldsymbol{J}\rangle$, we obtain $V = -\frac{e^{-jkL}}{2W}\int_{-W}^{W}\int_{-L}^{L} cos\left(\frac{\pi z}{2L}\right)\mathrm{d}z\mathrm{d}x = -\frac{4L}{\pi}e^{-jkL} = j\frac{\lambda}{\pi}$ Volt. The equivalent circuit of the receiving antenna is formed by a Thèvenin voltage source (=open circuit voltage) with internal impedance $\frac{1}{0.01} = 100$ Ohm. The complex-valued voltage across the load becomes $V_{\mathrm{L}} = V * \frac{50}{100+50} = j\frac{\lambda}{3\pi}$.

3.6 An identical pair of antennas is facing each other with a separation distance $r = 10\lambda$ and are also polarization matched. The measured 50-Ohm S_{11} of the transmitting antenna is 0.1, while $S_{21} = j0.5$. Receiving antenna 2 is matched terminated (no reflection at the output port). Use the two-port S parameters together with Friis' transmission formula to resolve the antenna gain of one such antenna. (5p)

Answer:

Friis' formula is

$$\frac{P_r}{P_t} = \left(\frac{\lambda}{4\pi r}\right)^2 G_{0t}G_{0r}$$

where for identical antennas $G_{0t} = G_{0r} = G_0$. Further, assuming an incident power (or maximum available power) P_{inc}, the transmitted power $P_t = P_{inc}(1 - |S_{11}|^2)$. The received power is $P_r = P_{inc}|S_{21}|^2$, hence

$$\frac{|S_{21}|^2}{(1 - |S_{11}|^2)} = \left(\frac{1}{40\pi}\right)^2 G_0^2$$

Thus

$$G_0 = \sqrt{\frac{|S_{21}|^2(40\pi)^2}{(1-|S_{11}|^2)}} = \sqrt{\frac{25*40*40*\pi*\pi}{99}} \approx 63$$

which is 18 dBi.

4.0 Gapwave slot antenna array (25p)

Fig. 4.0(a) shows an 8×8 gap waveguide slot antenna array at 60 GHz (developed at the Antenna Systems Division, Chalmers), whose radiating part can be modelled by an 8×8 slot array over an infinitely large ground plane with element spacing $dx = dy = 4.5$ mm and a rectangular slot of length $L = 3$ mm and width $W = 0.4$ mm, as shown in Fig. 4.0(b). The coordinate system is defined as in the figure, with the origin at the centre of the array.

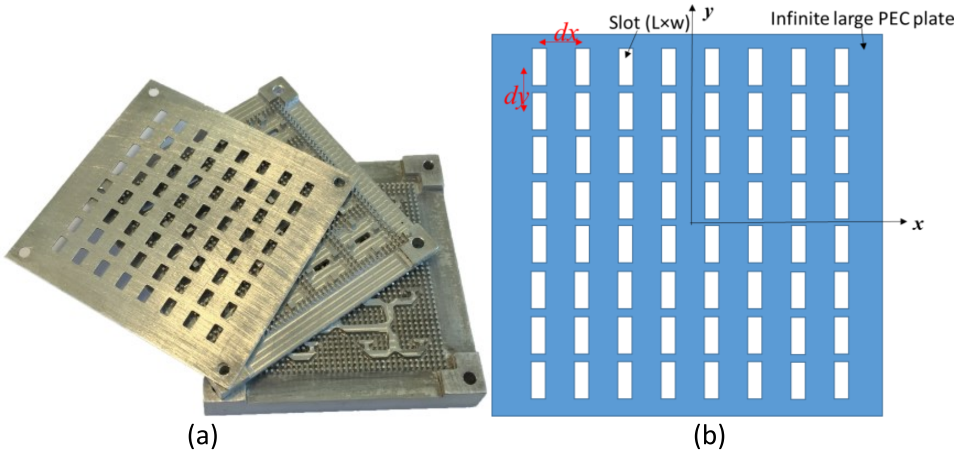


Fig. 4.0 Gap waveguide slot array antenna: (a) a photo, and (b) a simple model of the 8x8 slot array on an infinitely large PEC plate.

4.1 Write out the expression of the field distribution over a single slot at 60 GHz. The E-field amplitude is E_0. (3p)

Answer:

$$\boldsymbol{E}_{slot}(x', y') = \begin{cases} E_0 \cos\left(\frac{\pi}{3} y'\right) \hat{x}, |\, x'| \leq 0.2, |\, y'| \leq 1.5 \\ 0, \text{others} \end{cases}$$

4.2 Sketch the analysis procedure by writing out the methods and corresponding equivalence principle to use for analysing this slot array. (4p)

Answer:

1) Use PEC equivalent principle to have magnetic current and close the slot with PEC. 2) Use imaging to remove the ground plane and magnetic currents get 2 times. Calculate the element far field function. 3) Use array theory to get array factor. 4) The final far field function of the antenna is the production of element far field function and the array factor.

4.3 Write out the isolated far field function of single slot at 60 GHz. (3p)

Answer:

$$\boldsymbol{G}_{slot}(\theta, \varphi) = 2E_0 w \boldsymbol{G}_{img}(\theta, \varphi) \widetilde{M}(k\hat{y} \cdot \hat{r})$$

$$\boldsymbol{G}_{img}(\theta, \varphi) = C_k \hat{x} \times \hat{r}$$

$$\widetilde{M}(k\hat{y} \cdot \hat{r}) = \int_{-1.5}^{1.5} \cos(\frac{\pi y'}{3}) e^{jky'\hat{y} \cdot \hat{r}} dy'$$

The questions below are under the assumption that all mutual couplings between the slots can be neglected in order to simplify the solutions.

4.4 Now all slots are excited with the same excitation (the same amplitude and the same phase). Calculate the directivity, beamwidth, the first sidelobe level (relative to the level of the main beam) of the array at 60 GHz. State if you have any grating lobes. If there are some grating lobes, calculate the direction and the level relative to the level of the main beam. (5p)

Answer:

$\lambda = 5$ mm at 60 GHz so there is no grating lobe since the element spacing $dx = dy = 4.5 < 5$. The directivity is defined by array factor

$$D_0 = \frac{4\pi}{5^2}(8 \cdot 4.5)^2 = 28.14\text{dBi}$$

$$\theta_{3dB} = 2\arcsin\left(\frac{0.445 \cdot 5}{8 \cdot 4.5}\right) = 7.1^o$$

First sidelobe level = -13.2 dB.

4.5 All slots are excited with the same excitation (the same amplitude and the same phase). Calculate the directivity, beamwidth, the first sidelobe level of the array at 70 GHz. State if you have any grating lobes. If there are some grating lobes, calculate the direction and the level relative to the level of the main beam. (5p)

Answer:

$\lambda = 4.2857$ mm at 70 GHz so

$$D_{max} = \frac{4\pi}{4.28^2}(8 \cdot 4.5)^2 = 29.48 \text{ dBi}$$

And there are grating lobes since the element spacing $dx = dy = 4.5 > 4.2857$. There are 4 grating lobes:

$$(\theta_{1,0}, \varphi_{1,0}) = \left(\arcsin\left(\frac{4.2857}{4.5}\right), 0\right) = (72.25°, 0°)$$

$$(\theta_{-1,0}, \varphi_{-1,0}) = (72.25°, 180°)$$

$$(\theta_{0,1}, \varphi_{0,1}) = (72.25°, 90°)$$

$$(\theta_{0,-1}, \varphi_{0,-1}) = (72.25°, 270°)$$

The level of the two grating lobes in E-plane is the same as the main beam. The level of the two grating lobes in H-plane can be calculated by the element radiation pattern of the slot since the array factor makes the grating lobe's level the same as the main beam. From Fig. 5.22, we see the slot radiation pattern in H-plane is a cos function approximately. So the grating lobe in H-plane has the level of $\cos(72.25°)$ so

$$e_{grating} = \frac{1}{1 + 2 \times \frac{\cos 0°}{\cos 72.25°} + 2 \times \frac{\cos 72.25°}{\cos 72.25°}} = 0.1046 = -9.8dB$$

$$D_0 = e_{grating} \cdot D_{max} = -9.8 + 29.48 = 19.68\text{dBi}$$

The beam width and the first sidelobe are the same as before

$$\theta_{3dB} = 2\arcsin\left(\frac{0.445 \cdot 4.2857}{8 \cdot 4.5}\right) = 6.07^o$$

First sidelobe level = -13.2 dB.

4.6 Now due to some fault in feeding network, the slot excitations become those as shown in Fig.4.1: the slots in the 1st, 3rd, 5th, and 7th rows have the excitation amplitude of 1 *V*, the slots in the 2nd, 4th, 6th and 8th rows have the excitation amplitude of 0.707 *V,* but all slot excitations have the same phase. Calculate the directivity, beamwidth, the first sidelobe level of the array at 60 GHz. State if you have any grating lobes. If there are some grating lobes, calculate the direction and the level of the grating lobe(s) relative to the level of the main beam. (5p)

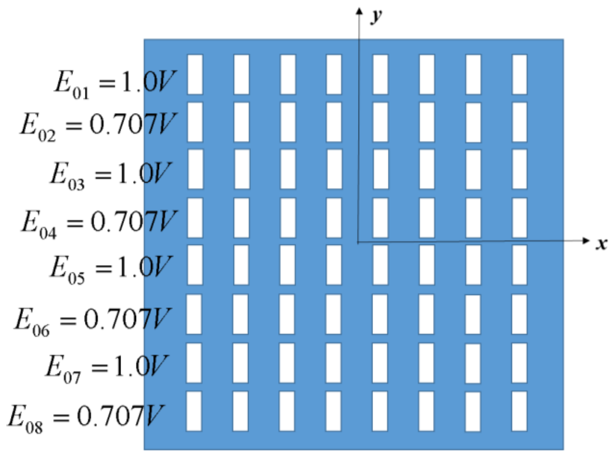
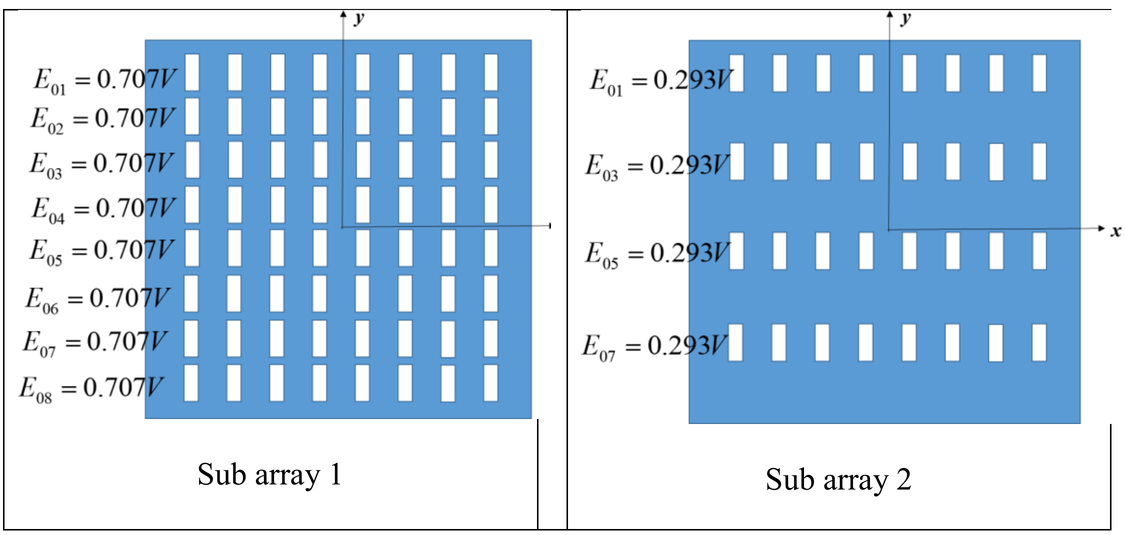


Fig. 4.1 Excitations of the slots when there is a fault in feeding network

Answer:

Now we use two-sub-array method. See figure below.



For sub-array 1, there is no grating lobe. So the gain of the sub-array 1 is

$$G_{01} = \frac{0.707 \cdot 8 \cdot 8}{0.707 \cdot 4 \cdot 8 + 1 \cdot 4 \cdot 8} D_{max} = 0.8284 D_{max}$$

For sub-array 2, there are 2 grating lobes in H-plane:

$$(\theta_{0,1}, \varphi_{0,1}) = \left(\arcsin\left(\frac{5}{4.5 \cdot 2}\right), 90°\right) = (33.75°, 90°)$$

$$(\theta_{0,-1}, \varphi_{0,-1}) = \left(\arcsin\left(\frac{5}{4.5 \cdot 2}\right), 270°\right) = (33.75°, 270°)$$

$$e_{grating,subarray2} = \frac{1}{1 + 2 \times \frac{\cos 33.75°}{\cos 33.75°}} = 0.33$$

$$G_{02} = \frac{0.293 \cdot 4 \cdot 8}{0.707 \cdot 4 \cdot 8 + 1 \cdot 4 \cdot 8} e_{grating,subarray2} D_{max}$$
$$= 0.17 * 0.33 D_{max} = 0.056 D_{max}$$

$G_{0,total} = G_{01} + G_{02} = (0.83 + 0.056) D_{max} = -0.53 dB$ +28.14dBi=27.61 dBi

The beam width and the first sidelobe are the same as before

$$\theta_{3dB} = 2\arcsin\left(\frac{0.445 \cdot 5}{8 \cdot 4.5}\right) = 7.1^{o}$$

First sidelobe level = -13.2 dB.

The grating lobes level is $10 * \log_{10}\left(\frac{0.05 * \cos 33.75°}{0.8284}\right) = -12.99\text{dB}$