Name:

Exam in the course Antenna Engineering 2014-06-02

ANTENNA ENGINEERING (SSY100)
(E4) 2013/14 (Period IV)

Monday 2 June 1400-1800 hours.

Teachers: Prof. Per-Simon Kildal, Associate Prof. Jian Yang, Associate Prof. Marianna Ivashina, Assistant Prof. Rob Maaskant.

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The exam consists of 2 parts. Part A is printed on colored paper and must be solved without using the textbook. When you have delivered the colored text and the solutions of Part A (latest 17:00), the textbook can be used for Part B of the exam.

You are allowed to use the following:
For Part A: **Pocket calculator of your own choice**
For Part B only: **Mathematical tables including Beta**
Pocket calculator of your own choice
Kildal's compendium "Foundations of Antennas: A Unified Approach for LOS and Multipath"
(The textbook can contain own notes and marks on its original printed pages. No other notes are allowed.)

Tentamen består av 2 delar. Del A har tryckts på färgade papper och skall lösas utan att använda läroboken. När du har inlämnat dom färgade arken med uppgifterna för del A och dina svar på dessa uppgifter (senast 17:00), kan du ta fram läroboken för att lösa del B.

Tillåtna hjälpmedel:
För del A: Valfri räknedosa
För del B: Matematiska Tabeller inkluderad Beta
Valfri räknedosa
Kildals lärobok "Foundations of Antennas: A Unified Approach for LOS and Multipath"

(Boken kan innehålla egna noteringar skrivna på de inbundna sidorna. Extra ark med noteringar tillåts inte.)

Name:

PART A (must be delivered before textbook can be used)

1.0 Foundations of Antenna Engineering (35p)

1.1. An antenna located at $\boldsymbol{r}_0$ in a coordinate system has the far field function $\boldsymbol{G}(\theta,\varphi)$. Write the far field function of the antenna when it is moved to the location $\boldsymbol{r}_1$. (2p)

A:

$$\boldsymbol{G}_1(\theta,\varphi) = \boldsymbol{G}_0(\theta,\varphi)e^{jk(\boldsymbol{r}_1-\boldsymbol{r}_0)\cdot\hat{\boldsymbol{r}}}$$

1.2. What is the phase centre of an antenna? (2p)

A:

The phase centre is the particular phase reference point which minimizes the phase variation of the co-polar far-field function over a given solid angle of interest.

1.3. What is the definition of BOR_0 and BOR_1 antennas? Give two examples for both BOR_0 and BOR_1 antennas (Sketch them for clarification). (2p)

A:

BOR_0 and BOR_1 antennas are antennas with a Body-of-revolution geometry and far field function with no φ variation and first order φ variation (cos φ or sin φ), respectively.

BOR_0 antennas: 1) dipole on z-axis; 2) small loop antenna in x-y plane.
BOR_1 antennas: 1) incremental dipole on y-axis; 2) conical horn antenna excited by TE11 waveguide mode.

1.4. We can characterize an antenna system by using the figure of merit G/T_{syst}. Explain what G and T_{syst} stand for. What are the main contribution factors to both G and T_{syst}? (List at least 3 factors for both G and T_{syst}.) (3p)

A:

G is the antenna gain and T_{syst} is the antenna system noise temperature.

Contribution factors to G: 1) aperture efficiency of the antenna; 2) radiation efficiency of the antenna; 3) mismatch factor of the antenna input with receiver; 4) alignment polarization efficiency.

Contribution factors to T_{syst}: 1) receiver's noise temperature; 2) ohmic losses of the antenna; 3) antenna's physical temperature; 4) relative power hitting the ground; 5) brightness temperature in the main beam direction.

1.5. In the course, three different methods for measuring antenna gain are discussed. Write two of them and list the requirements for these two methods. (2p)

A:

1) Two antenna method: the two antennas should be identical and the distance between antennas should be known.

2) Three antenna method: the distance between antennas should be known.
3) Replacement method: you need to have an antenna with known Gain.

1.6. What environment does a Reverberation Chamber emulate? Explain why the chamber can emulate it. (2p)

A:

Reverberation Chamber emulates the RIMP (Rich Isotropic Multipath environment.

Reverberation Chamber has many modes which can be stirred by different means in the chamber. Each mode can be considered as 8 incident waves with different incident angles.

1.7. List three antenna characteristics that can be measured using a Reverberation Chamber, and the advantages or disadvantages compared to the traditional anechoic chamber measurement. (3p)

A:

Radiation efficiency, diversity gain, total radiate power, receiver sensitivity and data throughput.

Faster, cheaper, simpler with similar or better accuracy compared to anechoic chamber measurement.

1.8. Antennas can be analyzed in terms of three incremental elementary sources. Write the name of them and the expression of the far field function due to the three incremental sources with *y* polarization. (3p)

A:

a) Incremental electric current source

$$G_{id}(\theta,\varphi) = C_k[\hat{\boldsymbol{y}} - (\hat{\boldsymbol{y}}\cdot\hat{\boldsymbol{r}})\hat{\boldsymbol{r}}] = C_k\left[\cos\theta\sin\varphi\hat{\boldsymbol{\theta}} + \cos\varphi\hat{\boldsymbol{\varphi}}\right]$$

b) Incremental magnetic current source

$$G_{im}(\theta,\varphi) = C_k[-(\hat{\boldsymbol{x}})\times\hat{\boldsymbol{r}}] = C_k\left[\sin\varphi\hat{\boldsymbol{\theta}} + \cos\theta\cos\varphi\hat{\boldsymbol{\varphi}}\right]$$

c) Incremental Huygen’s source

$$G_H(\theta,\varphi) = 2C_k\cos^2(\frac{\theta}{2})\left[\sin\varphi\hat{\boldsymbol{\theta}} + \cos\varphi\hat{\boldsymbol{\varphi}}\right]$$

1.9. What is the directivity limitation (upper bound) of a large antennas? Assume that the physical area of the antenna is *A*. (2p)

A:

$$D_{max} = \frac{4\pi}{\lambda^2}A$$

1.10. What is the directivity limitation (upper bound) for small antennas? (1p)

A:

$$D_{max} = 3$$

1.11. Write the value of the directivity for the following small antennas: (i) a halfwave dipole in free space; (ii) a quarter-wave monopole on a ground plane; (iii) a resonant slot on infinite larger ground plane. (3p)

A:

(i) 2.16 dBi; (ii) 5.16 dBi; (iii) 5.16 dBi.

1.12. During the course, two classes of aperture antennas have been analyzed in the Chapter "Radiation from apertures". Describe these two classes of apertures and how they can be analyzed using aperture theory (i.e., by using which equivalent principle and incremental source)? (3p)

A:

i) Apertures in PECs. Using PEC equivalent and magnetic current to analyze.

ii) Apertures in free space. Using free space and Huygens equivalent to analyze. Use both electric and magnetic currents.

1.13. What is the 1st sidelobe level compared to the beam maximum for both rectangular and circular uniform apertures? (2p)

A:

i) - 13.2 dB;

ii) -17.6 dB.

1.14. Describe the condition for non-radiating grating lobes for planer array antennas. (2p)

A:

One of the following three alternative solutions is OK.

i) $\sqrt{(\sin\theta_0\cos\varphi_0 + p_x\frac{\lambda}{d_x})^2 + (\sin\theta_0\sin\varphi_0 + q_x\frac{\lambda}{d_y})^2} > 1 + \frac{\lambda}{D}$

ii) $d_x < \frac{\lambda}{1+|\sin\theta_0|+\lambda/D_x}$; $d_y < \frac{\lambda}{1+|\sin\theta_0|+\lambda/D_y}$

iii) $d_x < \frac{\lambda}{2}$; $d_y < \frac{\lambda}{2}$

1.15. Write the expression of the directivity of a rectangular planer array ($L_x \times L_y$) with the main beam at (θ_0, φ_0). (3p)

A:

$$D = e_{\mathrm{grt}}e_{\mathrm{pol}}e_{\mathrm{ill}}\cos\theta_0 D_{max}$$

where

e_{grt} is the grating lobe efficiency, e_{pol} is the polarization efficiency, e_{ill} is the illumination efficiency, and

$$D_{max} = \frac{4\pi}{\lambda^2}L_xL_y$$

2.0 MIMO System and Capacity (15p)

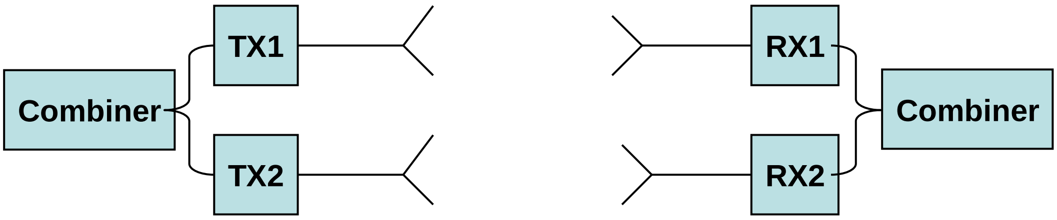


Fig 1: A MIMO system.

Fig. 1 shows a MIMO system in free-space whose antenna patterns/beams can be controlled arbitrarily. Assuming that the power at the transmitter side is divided equally, the capacity formula can be calculated as

$$C = \log_2 \left[\det \left(\boldsymbol{I} + \frac{\gamma_0}{N} \boldsymbol{H}\boldsymbol{H}^H \right) \right],$$

where the MIMO channel **H** is a $M \times N$ matrix, and where N is the number of transmitting antennas, M is the number of receiving antennas, γ_0 is the reference signal-to-noise ratio (SNR), "det" stands for determinant of a matrix, and the superscript ^{H} represents the Hermitian (transpose and conjugate) operator. Hint: $\det\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = ad - bc$.

2.1. What is "Capacity" of a communication system? (1p)
A:

Capacity is the maximum data rate a communication system can achieve with negligible error rate.

2.2. What is a MIMO system? What does "MIMO" stand for? (2p)
A:

MIMO stand for multi-input and multi-output. It is a system where there are more than one transmitting antennas and more than one receiving antennas. By transmitting more than one independent data stream through MIMO system, one can achieve higher data rate.

2.3. What are the benefits of using MIMO systems? (2p)
A:

Using MIMO to transmit independent multiple data streams (spatial multiplexing), one can enhance the data rate; using MIMO for diversity, one can achieve more reliable communication (or larger diversity gain).

2.4. RX1 receives signals only from TX1, and RX2 receive signal only from TX2. What is the MIMO capacity for the cases when: a) γ_0 = 0 dB, b) γ_0 = 10 dB? (3p)
A:

$$H = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$ a) $C_a = \log_2[\det(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix})] = 1.1699 \cdot$

b) $C_b = \log_2[\det(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \frac{10}{2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix})] = 5.1699 \cdot$

2.5. RX1 receives signals from both TX1 and TX2, with the same amplitudes and phases; and RX2 receives signals from both TX1 and TX2, with also the same amplitudes and phases. What is the MIMO capacity for the cases when a) γ_0 = 0 dB, b) γ_0 = 10 dB? (3p)

A:

$$H = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}.$$ a) $C_a = \log_2[\det(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix})] = 1.5850 \cdot$

b) $C_b = \log_2[\det(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \frac{10}{2}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix})] = 4.3923 \cdot$

2.6. Rx1 receives signals from both TX1 and TX2, with the same amplitudes but 90 degree phase shifted; and RX2 receives signals from both TX1 and TX2, with also the same amplitudes but 90 degree phase shifted. What is the MIMO capacity for the cases when a) γ_0 = 0 dB, b) γ_0 = 10 dB? (3p)

A:

$$H = \begin{bmatrix} 1 & j \\ j & 1 \end{bmatrix}.$$ a) $C_a = \log_2[\det(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 1 & j \\ j & 1 \end{bmatrix}\begin{bmatrix} 1 & -j \\ -j & 1 \end{bmatrix})] = 2 \cdot$

b) $C_b = \log_2[\det(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \frac{10}{2}\begin{bmatrix} 1 & j \\ j & 1 \end{bmatrix}\begin{bmatrix} 1 & -j \\ -j & 1 \end{bmatrix})] = 6.9189 \cdot$

2.7. Which case(s) gives the best performance in terms of capacity (for γ_0 = 0 dB and γ_0 = 10 dB cases, respectively)? (1p)

A:

Case 2.6) gives the best capacity for both γ_0 = 0 dB and γ_0 = 10 dB cases.

Name:

PART B (You can use the textbook to solve this problem, but only after PART A has been delivered)

3.0 Antenna Reciprocity, Equivalent Circuits of Antennas, Field Equivalence Principles, Boundary Conditions, Reaction Concept (30p)

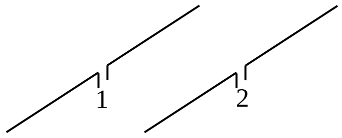


Fig 2: A pair of identical H-plane coupled dipole antennas.

3.1 Consider an identical pair of dipole antennas in a side-by-side arrangement, in free space (see Fig. 2). When antenna 1 is excited by 1 A, the *open-circuit* voltage of antenna 2 turns out to be $20e^{j\pi}$ Volt and the source voltage at port 1 is 50 Volt. Next, antenna 2 is excited by a voltage source of $20e^{j\pi}$ Volt. Compute the *short-circuited* current at antenna port 1 and the source current at port 2 using network theory. Hint: the inverse of a 2×2 matrix is: $\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$. (4p)

A: Use that $\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$, where $I_1 = 1$, $V_1 = 50$, $I_2 = 0$ (open circuit), and $V_2 = 20e^{j\pi}$. Hence, $Z_{11} = \frac{V_1}{I_1} = Z_{22} = 50\ \Omega$, $Z_{12} = Z_{21} = \frac{V_2}{I_1} = 20e^{j\pi}\ \Omega$. Next, we use the admittance matrix equation $\frac{1}{Z_{11}Z_{22}-Z_{21}Z_{12}}\begin{bmatrix} Z_{22} & -Z_{12} \\ -Z_{21} & Z_{11} \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$ to compute I_1 and I_2 for $V_2 = 20e^{j\pi}$and $V_1 = 0$ (short circuit), this yields $I_1 = -0.19$ A, and $I_2 = -0.48$ A.

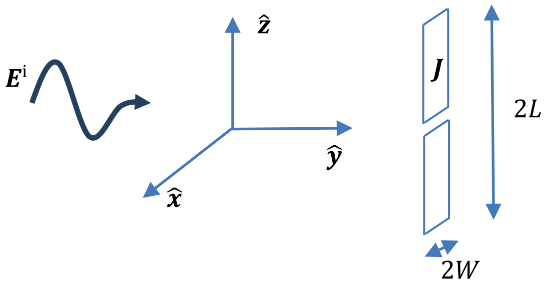


Fig 3. A strip dipole excited by an incident plane wave.

3.2 Consider Fig. 3, which shows a $\lambda/2$ strip dipole antenna placed at $y = L$. The surface current distribution, when excited by a current source $I = 1$ A across the infinitesimal strip gap at $z = 0$, is $\boldsymbol{J} = \frac{1}{2W}\cos\left(\frac{\pi z}{2L}\right)\hat{\boldsymbol{z}}$ A/m with finite support $-L \leq z \leq L$ and $-W \leq x \leq W$. The antenna self-admittance is 0.01 Siemens. Given these transmitting characteristics, consider now the receiving situation, where this antenna is excited by the plane wave field $\boldsymbol{E}^{\mathrm{i}} = e^{-jky}\hat{\boldsymbol{z}} - 0.5e^{-jky}\hat{\boldsymbol{x}}$, where k is the free-space wavenumber.

(a) Is the incident field circularly polarized, linearly polarized, or elliptically polarized? (1p)

A: Linearly polarized

(b) Compute the open-circuit voltage at the center of the antenna through a reaction integral and show in steps that it equals $V = -\frac{4L}{\pi}e^{-jkL}$ Volt. (3p)

A: Use the reaction integral formula, Eq. (4.93) from the book, i.e., $V = -\frac{1}{I}\langle \boldsymbol{E}^{\mathrm{i}}, \boldsymbol{J}\rangle$. This yields $V = -\frac{e^{-jkL}}{2W}\int_{-W}^{W}\int_{-L}^{L} cos\left(\frac{\pi z}{2L}\right)\mathrm{d}z\mathrm{d}x = -\frac{4L}{\pi}e^{-jkL}$ Volt.

(c) Compute the received power when the antenna is terminated by a 50 Ohm load using the open-circuit voltage given in (b). (3p)

A: The equivalent circuit of the receiving antenna is formed by a Thèvenin voltage source (=open circuit voltage) with internal impedance $\frac{1}{0.01} = 100$ Ohm. The current through the load $I_{\mathrm{L}} = V/(100 + 50)$. The dissipated power in the load is therefore $P_{\mathrm{L}} = \frac{1}{2}Re(|I_{\mathrm{L}}|^2 50) = \frac{4}{225}\left(\frac{L}{\pi}\right)^2$ Watts.

(d) Compute the gain in dBi (numeric value) in the direction of the plane wave. (5p)

A: There are several ways to do this. However, the simplest way is **not** through the radiation integral but through formula (2.129) from the book which gives a relation for the open-circuit voltage V (which we already know, see question 1.2) and the far-field function. That is, $V = -\frac{2j\lambda}{\eta I}G_z E_z^{\mathrm{i}}$, with $V = -\frac{4L}{\pi}e^{-jkL}$, $E_z^{\mathrm{i}} = e^{-jkL}$, and $I = 1$, from which we find the far field function value $G_z = \frac{2L\eta}{\pi j\lambda}$. The power radiated per solid angle in the direction of the plane wave is $P_{\mathrm{rad}} = \frac{1}{2\eta}|G_z|^2$ W/Sr, and the total radiated power is $P_{\mathrm{tot}} = \frac{1}{2}\mathrm{Re}(Z_{11})|I|^2 = \frac{\mathrm{Re}(Z_{11})}{2} = 50$ Watt. The gain is therefore: $\mathrm{Gain} = 10\log_{10}(4\pi P_{\mathrm{rad}}/P_{\mathrm{tot}}) = 10\log_{10}\left(\frac{4\eta}{25\pi}\left(\frac{L}{\lambda}\right)^2\right) = 10\log_{10}(1.2) = 0.8$ dBi.

(e) What could, in general, be the reason for the low gain that was found in (d) relative to the standard gain of a $\lambda/2$ dipole antenna found in most text books? (1p)

A: Ohmic losses generally reduce the gain. Indeed, the real part of the input impedance is rather high, i.e. 100 Ohm (normally one expects a value of around 75 Ohm). So, the remaining loss resistance of approximately 25 Ohm has led to a reduced gain.

3.3 Explain whether or not the mutual coupling can be neglected between a transmitting antenna and a receiving antenna, the latter of which is placed in the far field zone of the transmitting antenna and is known to receive the transmitted signal. (2p)

A: The mutual coupling cannot be neglected, otherwise we would not receive any signal. In fact, based on the two-port matrix description for a pair of antennas we have that the open-circuited Thèvenin voltage source of antenna 2 is $V_2 = Z_{21}I_1$, so if $Z_{21} = 0$ (no mutual coupling), then $V_2 = 0$, irrespective of the value of I_1 at the port of the transmitting antenna.

3.4 A microstrip patch antenna in air dielectrics is at resonance and has a width of $\lambda/4$.

(a) Explain whether the physical patch length is shorter, equal, or longer than $\lambda/2$? (2p)

A: Shorter, because the fringing fields help extending the patch slightly.

(b) Explain by using the magnetic current source model and the image principle for computing the far-field pattern of patch antennas what the approximate broadside gain of the patch antenna will be. Is it likely to be in the order of $1-2$ dBi, $2-5$ dBi, or $7-10$ dBi? (3p)

A: The magnetic line current for modeling the fringing fields has an approximate gain of around 2 dBi, but we have two of these line currents thus we add 3 dB, and we have a ground plane so we add another 3 dB, which gives us approximately 8 dBi. The approximate range is therefore at least 6 dBi, thus $7-10$ dBi.

3.5 Consider an x-polarized incremental electric current source of unit magnitude, i.e., $\eta I_0 l = 1$, placed at an height h above an infinite PEC ground plane at $z = 0$. The dipole is in the far-field of the ground plane. Derive an analytical expression for the induced electric current in the PEC ground plane (in Cartesian coordinates). Hint: compute the dipole radiated field at the ground plane and find the physical equivalent electric current by applying appropriate boundary conditions. (5p)

A: The H-field of the incremental dipole source in its far-field is given through Eq. (4.66) as: $\boldsymbol{H}^{\text{dip}} = \frac{C_k}{\eta}(\hat{\boldsymbol{r}} \times \hat{\boldsymbol{x}})\frac{1}{r}e^{-jkr}$. With the dipole at height h, we have that the observation point at the ground plane at $z = 0$, seen from the dipole, is given as $\boldsymbol{r} = x\hat{\boldsymbol{x}} + y\hat{\boldsymbol{y}} - h\hat{\boldsymbol{z}}$, so that $r = \sqrt{x^2 + y^2 + h^2}$, and $\hat{\boldsymbol{r}} = \boldsymbol{r}/r$. The H-field at the ground plane is therefore: $\boldsymbol{H}^{\mathrm{dip}}(z=0) = -\frac{C_k(h\hat{\boldsymbol{y}}+y\hat{\boldsymbol{z}})}{\eta(x^2+y^2+h^2)}e^{-jk\sqrt{x^2+y^2+h^2}}$. Using the image principle, we have at the ground plane the total H-field $\boldsymbol{H}_{\mathrm{tot}}(z=0) = -\frac{2C_k h\hat{\boldsymbol{y}}}{\eta(x^2+y^2+h^2)}e^{-jk\sqrt{x^2+y^2+h^2}}$. The PEC equivalent current is given through (4.61) as $\boldsymbol{J} = \hat{\boldsymbol{n}} \times \boldsymbol{H}_{\mathrm{tot}} = \hat{\boldsymbol{z}} \times \boldsymbol{H}_{\mathrm{tot}} = \frac{2C_k h\hat{\boldsymbol{x}}}{\eta(x^2+y^2+h^2)}e^{-jk\sqrt{x^2+y^2+h^2}}$.

3.6 Provide a reason why field equivalence principles are useful (1p).

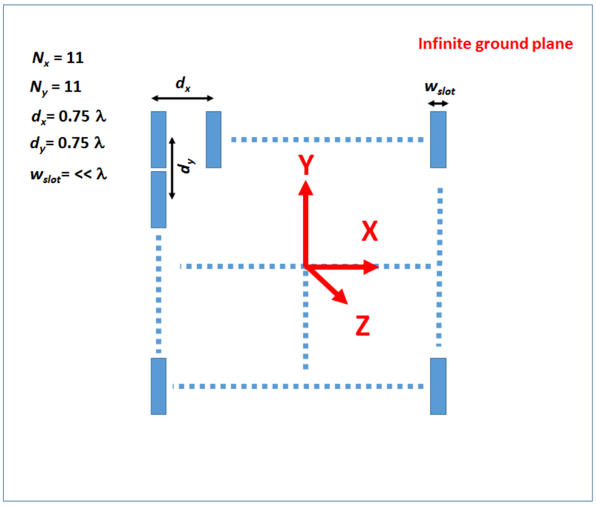
A: The problem may be easier to solve when formulating an equivalent field problem.

Name:

4.0 Design of a Planar Array (20p)

We have an *x*-polarized planar slot array antenna with the aperture area of $8\lambda \times 8\lambda$, where λ is the wavelength at the operating frequency. The element spacing is equal to 0.75λ in both the *x*- and *y*-directions. The elements are resonant slots on an infinite ground plane. Assume that the magnitudes of the excitation coefficients are the same.

4.1 Sketch your design of the slot planar array along with your coordinate system. (2p)



4.2 Write the expression of the far field function of the isolated single slot element on the infinite ground plane. (2p)

From Eq. 5.82, $\boldsymbol{G}_{slt}(\hat{\boldsymbol{r}}) = 2E_o\omega \boldsymbol{G}_{img}(\hat{\boldsymbol{r}})\widetilde{M}(k\hat{\boldsymbol{y}} \cdot \hat{\boldsymbol{r}})$

the far-field function of the isolated single slot element is a product of three factors: (1) the factor $2E_o\omega$, (2) the far-field function $\boldsymbol{G}_{img} = C_k(\hat{\boldsymbol{y}} \times \hat{\boldsymbol{r}}) = C_k[\cos\varphi\hat{\theta} - \cos\theta\sin\varphi\,\hat{\varphi}]$ of a unit incremental y-directed magnetic current source and (3) the Fourier transform of the magnetic current distribution $\widetilde{M}(k\hat{\boldsymbol{y}} \cdot \hat{\boldsymbol{r}}) = \int_{-l/2}^{l/2} \cos(\pi y'/l) e^{jky'\hat{\boldsymbol{y}}\cdot\hat{\boldsymbol{r}}}\, dy'$.

4.3 The maximum radiation direction of the array should be at $(\theta_0, \varphi_0) = (30^o, 90^o)$. Calculate the linear phase progression of the element excitations. (3p)

The linear phase progression for the planar array can be computed from the expressions for the propagation constants of the phase excitation (Eq. 10.67 from the book):

$$k_{\Phi_x} = -\frac{\Delta\Phi_x}{d_x} \quad \text{and} \quad k_{\Phi_y} = -\frac{\Delta\Phi_y}{d_y}$$

and the relationships between the propagation constants and the maximum radiation direction (Eq. 10.75 in the book):

$$\sin\theta_o = \sqrt{\left(k_{\Phi_x}\right)^2 + \left(k_{\Phi_y}\right)^2}/k \qquad \text{and} \qquad \tan\phi_o = k_{\Phi_y}/k_{\Phi_x}.$$

Since $\varphi_0 = 90^o$ and $\theta_0 = 30^o$, $k_{\Phi_x} = 0$ and $k_{\Phi_y} = k\sin\theta_o$, and hence $\Delta\Phi_x = 0$ and $\Delta\Phi_y = -0.75\pi = -2.36\,[rad] = -135\,^o$.

4.4 Calculate the directivity of the array antenna and its 3dB beamwidths in both the E- and H-planes. (3p)

The directivity of the array antenna can be determined by using the following formula (Eq. 10.87 from the book):

$$D = \epsilon_{grt}\cos\theta_o\epsilon_{pol}\epsilon_{ill}D_{max},$$

where the maximum available directivity can be found from the given element spacing and the number of elements N=11.

$$D_{\max} = \frac{4\pi}{\lambda^2}A = \frac{4\pi}{\lambda^2}(Nd)^2 = 29.32\ [\text{dBi}]\ .$$

Since the magnitudes of the excitation coefficients are the same, the aperture illumination efficiency can be assumed to be equal to 1. Also, the polarization efficiency can be assumed to be equal to 1 since the slots are infinitely thin and in the plane of scan do not exhibit significant cross-polarization. From the given direction of maximum radiation, we can compute $\cos\theta_o = \cos 30^o = 0.866 = -0.6247\ [\text{dBi}]$.

Now, the only unknown factor in the expression of the directivity is the grating-lobe efficiency. It can be computed from Eq. 10.91 in the book:

$\epsilon_{grt} = \frac{|G(\theta_o,\varphi_o)|^2}{\sum_{pq}|G(\theta_{pq},\varphi_{pq})|^2\frac{\cos\theta_o}{\cos\theta_{pq}}}$, where $G(\theta_o,\varphi_o) = AF(\theta_o,\varphi_o)G_{el}(\theta_o,\varphi_o)$ and

where $AF(\theta_o,\varphi_o)$ is the array factor in the direction of the main beam, and $G_{el}(\theta_o,\varphi_o)$ is the far-field function of the array antenna element.

Now, we will check whether the grating lobes exist in direction p by determining the grating lobe corresponding maxima:

$$\sin\theta_p\cos\varphi_p = \sin\theta_o\cos\varphi_o + p\frac{\lambda}{d_x}.$$

For $(\theta_0,\varphi_0) = (30^o, 90^o)$ and $p = \pm 1$ $(q = 0)$, $\cos\varphi_p = \cos 0 = 1$, and grating lobe appears at $\sin\theta_p = \frac{\pm 1}{0.75}$. Hence, there are no grating lobes in the visible region in direction p.

Similarly, we will check whether the grating lobes exist in direction q by determining the grating lobe corresponding maxima:

$$\sin\theta_q\sin\varphi_q = \sin\theta_o\sin\varphi_o + q\frac{\lambda}{d_y}.$$

For $(\theta_0, \varphi_0) = (30^o, 90^o)$ and $q = \pm 1$ $(p = 0)$, this becomes $\sin\theta_q = 1 + \frac{\pm 1}{0.75}$. $\sin\theta_q = -0.833$ corresponds to the grating lobe in the visible region. The maximum of this grating lobe is at $\theta_q = -56.44^o$.

Now, we can compute the grating lobe efficiency:

$$\epsilon_{grt} = \frac{|G(\theta_o, \varphi_o)|^2}{|G(\theta_o, \varphi_o)|^2 + \Sigma_{pq}|G(\theta_{pq}, \varphi_{pq})|^2 \frac{\cos\theta_o}{\cos\theta_{pq}}} = \frac{1}{1 + \frac{\cos\theta_o}{\cos\theta_{pq}}} = 0.39 = -4.09\ [dBi],$$

where we have assumed that the far-field function of the array antenna element $G_{el}(\theta_o, \varphi_o)$ is an omni-directional function.

By using the results obtained above, we can find the directivity of the planar array

$$D = (-4.09 + 0 + 0 - 0.6247 + 29.32) = 24.6\ [dBi]$$

The 3dB beamwidths in the E- and H-planes can be obtained from the approximate formulas for the rectangular aperture antennas, where we have accounted for the beamwidth factor $1/\cos\theta_o$ increase in the plane of scanning (see Eq. 10.67 in the book):

In the E-plane $(\varphi = 0^o)$: $\Delta\theta_{3dB} = 2\,arcsin\left(\frac{0.445\lambda}{Nd}\right) = 6.2^0$

In the H-plane $(\varphi = 90^o)$: $\Delta\theta_{3dB} = 2\,arcsin\left(\frac{0.445\lambda}{Nd\cos(30^o)}\right) = 7^0$

4.5 Now the directivity does not fulfil the system specification requirements. The directivity should be improved. Without changing the size of the array, please re-design your array, and re-calculate the directivity of your newly designed array. (3p)

Since the size of the array and specified scan directions are fixed, we can only change the element spacing.

To determine the required element spacing that assures no grating lobes when scanning in direction $(\theta_0, \varphi_0) = (30^o, 90^o)$, we can use the earlier given formula the grating lobe corresponding maxima

$$\sin\theta_q \sin\varphi_q = \sin\theta_o \sin\varphi_o + q\frac{\lambda}{d_y}.$$

For $q = -1$, $\sin\varphi_q = 90^o$, $\sin\theta_o = 30^o$ and $\sin\theta\varphi_o = 90^o$, we have $-\sin\theta_q = \sin\theta_o - \frac{1}{d_y}$. Hence, for no grating lobes, we need to satisfy the following conditions: $\frac{1}{2} - \frac{1}{d_y} \geq 1$. This leads to $d_y \leq 0.67\lambda$.

Alternatively, one can determine the required element spacing that assures no grating lobes in the visible region, we can use the following formulas

$$d_y \leq \frac{\lambda}{1 + |\sin\theta_o| + \lambda/D}$$

For $\theta_0 = 30^o$, this becomes $d_y \leq 0.62\lambda$

4.6 Now the maximum radiation direction should be steered to $(\theta_0, \varphi_0) = (30^o, 45^o)$. Calculate the linear phase progression of the element excitations. (4p)

From Eq. 10.67 and Eq. 10.75 in the book, we can calculate the linear phase progression corresponding to the maximum radiation direction (θ_0, φ_0).

$$k_{\Phi_x} = -\frac{\Delta\Phi_x}{d_x} \quad \text{and} \quad k_{\Phi_y} = -\frac{\Delta\Phi_y}{d_y}$$

where

$$\sin\theta_o = \sqrt{\left(k_{\Phi_x}\right)^2 + \left(k_{\Phi_y}\right)^2}/k \qquad \text{and} \qquad \tan\phi_o = k_{\Phi_y}/k_{\Phi_x}.$$

Since $\varphi_0 = 45^o$ and $\theta_0 = 30^o$, $\sin\theta_o = 1/2$ and $\tan\phi_o = 1$. Hence,
$k_{\Phi_x} = k_{\Phi_y} = \frac{k}{2\sqrt{2}}$,
and $\Delta\Phi_x = \Delta\Phi_y = -d_y k_{\Phi_y} = -0.75\frac{\pi}{\sqrt{2}} = -1.66\ [rad] = -95\ ^o$.

4.7 Explain how to determine the embedded element impedance, the scan impedance, and the isolated element impedance of an antenna array element. (3p)

The embedded element impedance is determined when the other array elements are in place and match terminated, the scan impedance is determined when the other array elements are in place and excited, and the isolated element impedance is determined when the other elements are not present.