Transformer, Signaler & System, D3
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1a)

$$x(t+T) = e^{j(13(t+T)+\pi)} = e^{j(13t+\pi)} \cdot e^{j13T} = x(t) \; ?$$

$$e^{j13T} = 1 \quad \text{om} \quad 13T = 2\pi \qquad T = \frac{2\pi}{13}$$

Periodisk ! med perioden $T = \frac{2\pi}{13}$

b)

$$S = \sum_{n=-\infty}^{\infty} \sin\left(\frac{n\pi}{9}\right)\left(\delta[n] + \delta[n-3] + \delta[n-6]\right) =$$

$$= \sin\left(\frac{0 \cdot \pi}{9}\right) + \sin\left(\frac{3\pi}{9}\right) + \sin\left(\frac{6\pi}{9}\right) =$$

$$= 0 + \sin\left(\frac{\pi}{3}\right) + \sin\left(\frac{2\pi}{3}\right) = 2 \cdot \left(\frac{\sqrt{3}}{2}\right) = \sqrt{3}$$

c)

$$x[n] = x_1[n] * x_2[n]$$

$$x_1[n] = u[n-2] \overset{\mathcal{Z}}{\longleftrightarrow} \frac{z}{z-1} \cdot z^{-2} = X_1(z)$$

$$x_2[n] = \left(\frac{2}{3}\right)^n \cdot u[n] \overset{\mathcal{Z}}{\longleftrightarrow} \frac{z}{z-\frac{2}{3}} = X_2(z)$$

$$X(z) = X_1(z) \cdot X_2(z) = \frac{z \cdot z^{-2} \cdot z}{(z-1)(z-\frac{2}{3})} =$$

$$= \frac{1}{(z-1)(z-\frac{2}{3})} = \frac{1}{z^2 - z \cdot \frac{5}{3} + \frac{2}{3}}$$

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2. $y_s(t) = e^{-6t} \cdot u(t)$ Stegsvar

Laplace transf. $Y_s(s) = \frac{1}{s+6}$

Insignal $x'(t) = u(t) \xleftrightarrow{\mathcal{L}} X'(s) = \frac{1}{s}$

$$Y_s(s) = X'(s) \cdot H(s) = \frac{1}{s} \cdot H(s)$$

$$H(s) = s \cdot Y_s(s) = \frac{s}{s+6}$$ Överf. funktion !

Ny utsignal $y(t) = e^{-t} \cdot u(t) \xleftrightarrow{\mathcal{L}} Y(s) = \frac{1}{s+1}$

Sök insignal $x(t) \xleftrightarrow{\mathcal{L}} X(s)$

$$X(s) = \frac{Y(s)}{H(s)} = \frac{s+6}{(s+1)s} = \{\text{P.B.U.}\} = \frac{A}{s} + \frac{B}{s+1}$$

$s+6 = A(s+1) + B \cdot s$ $s^1: \; 1 = A + B$ | $A = 6$
$s^0: \; 6 = A$ | $B = -5$

$$X(s) = \frac{6}{s} - \frac{5}{s+1} \overset{\mathcal{L}^{-1}}{\Rightarrow} x(t) = \left(6 - 5e^{-t}\right) u(t)$$

b) $H(s) = \frac{s}{s+6} = \frac{Y(s)}{X(s)}$

$$Y(s)(s+6) = s\,X(s) \overset{\mathcal{L}^{-1}}{\Rightarrow} \frac{dy(t)}{dt} + 6y(t) = \frac{dx(t)}{dt}$$

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3. $y[n] + 0{,}5\,y[n-1] = x[n] - 2{,}1\,x[n-1]$

z - transformera

$$Y(z) + 0{,}5 \cdot z^{-1} Y(z) = X(z) - 2{,}1\, z^{-1} X(z)$$

$$Y(z)\left(1 + 0{,}5 z^{-1}\right) = X(z)\left(1 - 2{,}1 z^{-1}\right)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 - 2{,}1 z^{-1}}{1 + 0{,}5 z^{-1}} = \frac{z - 2{,}1}{z + 0{,}5}$$

$$x[n] = 0{,}8^n u[n] \overset{z}{\longleftrightarrow} X(z) = \frac{z}{z - 0{,}8}$$

$$Y(z) = H(z) X(z) = \frac{z(z-2{,}1)}{(z+0{,}5)(z-0{,}8)} = z \cdot \underbrace{\frac{z-2{,}1}{(z+0{,}5)(z-0{,}8)}}_{\text{P.B.U.}}$$

$$\frac{z-2{,}1}{(z+0{,}5)(z-0{,}8)} = \frac{A}{z+0{,}5} + \frac{B}{z-0{,}8}$$

$$z - 2{,}1 = A(z-0{,}8) + B(z+0{,}5)$$

z^1: $1 = A + B$ $\qquad B = 1 - A$

z^0: $-2{,}1 = -0{,}8A + 0{,}5B$ $\qquad -2{,}1 = -0{,}8A + 0{,}5(1-A)$

$\Rightarrow A = 2, \quad B = -1$

$$Y(z) = \frac{2z}{z+0.5} - \frac{z}{z-0.8}$$

Inv. z-transf.

$$y[n] = \left[2 \cdot (-0{,}5)^n - (0{,}8)^n\right] u[n]$$

4.

$$x(t) = \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \sin(n\omega_0 t)$$

$$\omega_0 = \frac{\pi}{L} = \frac{\pi}{\pi} \cdot 200 = 200 \quad r/s$$

Frekv. svar $\quad H(s)\Big|_{s=j\omega} = H(j\omega) = \dfrac{j\omega K}{600^2 - \omega^2 + j\omega 160}$

Krav vid $\omega = 3 \cdot \omega_0 \quad (n=3)$

$$\left| H(j\omega)\Big|_{\omega = 3\omega_0} \right| = 10$$

$$3\omega_0 = 600 \quad r/s$$

$$|H(j3\omega_0)| = \frac{3\omega_0 \cdot K}{\sqrt{(600^2 - 600^2)^2 + (3\omega_0 160)^2}} =$$

$$= \frac{3\omega_0 K}{3\omega_0 160} = 10 \quad \Rightarrow K = 1600$$

Svar: $K = 1600$

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5. $f_s = 800$ Hz
$N = 2^{10} = 1024$

$|X[k]|$ med "toppar" vid $k_1 = 64$ och $N-64 = 960$ svarar mot en reell sinusformad signal med frekvensen $f_1 = \frac{k_1}{N} \cdot f_s = \frac{64}{1024} \cdot 800 = 50$ Hz
("Nätverksbrum?")

$|X[k]|$ med "toppar" vid $k_2 = 128$ och $N-128 = 896$ svarar mot en reell sinusformad signal med frekvensen $f_2 = \frac{k_2}{N} \cdot f_s = \frac{128}{1024} \cdot 800 = 100$ Hz
("1:a övertonen till nätverksbrum?")

$G(s)$ med frekvenssvar $G(s)\big|_{s=j\omega} = G(j\omega)$ skall släcka ut $\omega_1 = 2\pi f_1$ och $\omega_2 = 2\pi f_2$

Täljarpolynom $T(j\omega)$ skall ha komplexkonjugerade nollställen $T(j\omega) = (j\omega - j\omega_1)(j\omega + j\omega_1)(j\omega - j\omega_2)(j\omega + j\omega_2)$

Vilket ger $T(s) = (s - j\omega_1)(s + j\omega_1)(s - j\omega_2)(s + j\omega_2) =$
$= (s^2 + \omega_1^2)(s^2 + \omega_2^2)$

$T(s) = s^4 + s^2(\omega_1^2 + \omega_2^2) + \omega_1^2 \omega_2^2 =$

$= s^4 + s^2 (2\pi)^2 \cdot 12500 + (2\pi)^4 \cdot 25 \cdot 10^6$

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1a) i) $g(t-t_0)$

ii) $g(t_0)\,\delta(t-t_0)$

iii) $g(-t_0)$

b) $u(t) = L\dfrac{di(t)}{dt} + Ri(t)$

Insignal	Utsignal
$i_1(t)$	$L\dfrac{di_1(t)}{dt} + Ri_1(t) = u_1(t)$
$i_2(t)$	$L\dfrac{di_2(t)}{dt} + Ri_2(t) = u_2(t)$
$ai_1(t)$	$L\dfrac{d}{dt}(ai_1(t)) + Rai_1(t) = au_1(t)$
$ai_1(t) + bi_2(t)$	$L\dfrac{d}{dt}(ai_1(t) + Rai_1(t) +$
	$+ L\dfrac{d}{dt}(bi_2(t) + Rbi_2(t) =$

$= au_1(t) + bu_2(t)$

Superposition!

Systemet är linjärt

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2)

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$$

$$x(t) = \delta(t) - e^{-3t}u(t) \overset{\mathcal{L}}{\longleftrightarrow} 1 - \frac{1}{s+3} = \frac{s+3-1}{s+3} = \overbrace{\frac{s+2}{s+3}}^{X(s)}$$

$$y(t) = \left(3e^{-5t} - e^{-3t}\right)u(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{3}{s+5} - \frac{1}{s+3} = Y(s)$$

$$Y(s) = \frac{3(s+3) - (s+5)}{(s+5)(s+3)} = \frac{3s+9-s-5}{(s+5)(s+3)} = \frac{2(s+2)}{(s+5)(s+3)}$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{2\cancel{(s+2)}}{(s+5)\cancel{(s+3)}} \cdot \frac{\cancel{(s+3)}}{\cancel{(s+2)}} = \frac{2}{s+5}$$

Impulssvar $h(t) = \mathcal{L}^{-1}\{H(s)\} = 2e^{-5t} \cdot u(t)$

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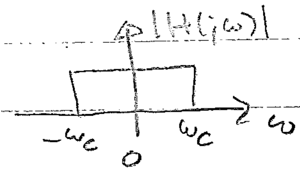
3/

$$H(j\omega) = \begin{cases} e^{-j\omega t_0} & , |\omega| \leq \omega_c \\ 0 & , |\omega| > \omega_c \end{cases}$$



$|H(j\omega)| = 1$, $\arg\{H(j\omega)\} = -\omega t_0$ för $|\omega| \leq \omega_c$

$$\omega_c = \frac{7\pi}{T} = \frac{7\pi}{2\pi} \cdot \omega_0 = 3{,}5\,\omega_0$$

$$x(t) \longrightarrow \boxed{H(j\omega)} \longrightarrow y(t)$$

a/ $$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t}$$ $c_0 = 0$, $c_k = \frac{2}{k^2}$, $k \neq 0$

$$x(t) \overset{FT}{\longleftrightarrow} X(j\omega) = \sum_{k=-\infty}^{\infty} 2\pi c_k \delta(\omega - k\omega_0)$$

$$Y(j\omega) = H(j\omega) \cdot X(j\omega) = \sum_{k=-3}^{3} 2\pi e^{-j\omega t_0} c_k \delta(\omega - k\omega_0) =$$

$$= 2\pi \sum_{k=-3}^{3} \cdot c_k e^{-jk\omega_0 t_0} \delta(\omega - k\omega_0)$$

$$y(t) = \sum_{k=-3}^{3} c_k^Y e^{+jk\omega_0 t} \quad \text{med } c_k^Y = c_k \cdot e^{-jk\omega_0 t_0}$$

b/ Medeleffekt $$\overline{P_y} = \frac{1}{T} \int_T |y(t)|^2 dt = \sum_{k=-3}^{3} |c_k^Y|^2 =$$

$$= \left\{ \omega_c = \frac{7\pi}{T} = \frac{7\pi}{2\pi} \cdot \omega_0 = 3{,}5\,\omega_0 \right\} = \sum_{k=-3}^{3} |c_k^Y|^2 = \{c_0 = 0\}$$

$$= 2\left(\frac{2}{1}\right)^2 + 2\left(\frac{2}{4}\right)^2 + 2\left(\frac{2}{9}\right)^2 = 8 + \frac{1}{2} + \frac{8}{81} = 8{,}56$$

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4)

$$x(t) = \cos(f_1 \cdot 2\pi t) + \cos(f_2 \cdot 2\pi t)$$

$$f_1 = 42 \text{ Hz} \quad , \quad f_2 = 105 \text{ Hz}$$

$$f_s = \frac{1}{T_s} = 336 \text{ Hz} \quad (\text{samplings frekvens})$$

a)

$$\Delta f = \frac{f_s}{N} = \frac{336}{64} = 5{,}25 \text{ Hz}$$

$$\Delta \omega = 2\pi \cdot \Delta f = 10.5\pi \text{ rad/s}$$

b) En reell sinusformad signal ger två "toppar" i DFT. Vi har två sinusformade signaler

Svar: 4 st

c)

$$\frac{k}{N} = \frac{f}{f_s} \; ; \; k_1 = N \cdot \frac{f_1}{f_s} = 64 \cdot \frac{42}{336} = 8$$

Även $N - 8 = 56$

$$k_2 = N \cdot \frac{f_2}{f_s} = 64 \cdot \frac{105}{336} = 20$$

Även $N - 20 = 44$

Svar: "Toppar" vid $k = 8, 20, 44$ och 56

d) Antal sampel per period $= \dfrac{\text{Signalens periodtid}}{\text{Sampelintervall}}$

$$x_1: \quad \frac{T_1}{T_s} = \frac{f_s}{f_1} = \frac{336}{42} = 8$$

$$x_2: \quad \frac{T_2}{T_s} = \frac{f_s}{f_2} = \frac{336}{105} = 3{,}2$$

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5/ $Y(z) = \frac{4z}{z^2+z+0{,}25} = \dots = \frac{4z}{(z+0{,}5)^2} = H(z)$

OBS! $Y(z)$ är överföringsfunktionen, vanligen $H(z)$.

a/ Från Beta: $\frac{az}{(z-a)^2} \xleftrightarrow{Z} na^n u[n]$

Låt $a = -0{,}5$

$$H(z) = \frac{1}{a} \cdot \frac{4a \cdot z}{(z-a)^2} = \frac{4}{a} \cdot \frac{az}{(z-a)^2}$$

och $h[n] = \frac{4}{-0{,}5} \cdot n \cdot (-0{,}5)^n \underset{u[n]}{\uparrow} = -8n(-0{,}5)^n u[n]$

b/ Stegsvar $s[n] \xleftrightarrow{Z} S(z)$

Insignal $u[n] \xleftrightarrow{Z} \frac{z}{z-1}$

$$S(z) = \frac{z}{z-1} \cdot H(z) = z \cdot \frac{4z}{(z-1)(z+0{,}5)^2}$$

$$\frac{S(z)}{z} = \frac{4z}{(z-1)(z+0{,}5)^2} = \frac{A}{z-1} + \frac{B}{(z+0{,}5)} + \frac{C}{(z+0{,}5)^2}$$

$$4z = A(z+0{,}5)^2 + B(z-1)(z+0{,}5) + C(z-1)$$

z^2: $0 = A + B$
z^1: $4 = A - 0{,}5B + C$
z^0: $0 = 0{,}25A - 0{,}5B - C$

Lös ekv. systemet

$\Rightarrow A = -B = \frac{16}{9}$

$C = \frac{4}{3}$

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/ forts 5

$$\frac{4}{3(-0{,}5)} \cdot \frac{(-0{,}5)z}{(z-(-0{,}5))^2}$$

$$S(z) = \frac{16}{9} \cdot \frac{z}{z-1} - \frac{16}{9} \cdot \frac{z}{z+0{,}5} + \underbrace{\frac{4}{3} \cdot \frac{z}{(z+0{,}5)^2}}$$

Inverstransformera!

$$s[n] = \frac{16}{9} \cdot u[n] - \frac{16}{9}(-0{,}5)^n u[n] + \frac{4}{3(-0{,}5)} \cdot n(-0{,}5)^n \cdot u[n]$$

$$s[n] = \left[\frac{16}{9}\left(1-(-0{,}5)^n\right) - \frac{8}{3} \cdot n(-0{,}5)^n\right] u[n]$$

$$s[n] = \frac{8}{9}\left[2\left(1-(-0{,}5)^n\right) - 3n(-0{,}5)^n\right] u[n]$$

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1a) Periodisk? Ja! $T = 2\,s$

b) $y[n] = v[n-2] = \ldots, 0, 0, 2, 0, \underline{3}, 1, 2, 0, 0, \ldots$

$\uparrow n=0$

2a) $$H(j\omega) = \frac{1}{\sqrt{1+\left(\frac{\omega}{\omega_c}\right)^4}}$$

$$\arg\{H(j\omega)\} = -\arctan\left\{\frac{\sqrt{2}\,\frac{\omega}{\omega_c}}{1-\left(\frac{\omega}{\omega_c}\right)^2}\right\}$$

3) a) i) 30 Hz
ii) $10 < f_c < 30$ Hz
iii) $f_c < 10$ Hz

b) i) 14 Hz
ii) $2 < f_c < 10$ Hz
iii) $f_c < 2$ Hz

4) $$Y(s) = \underbrace{\frac{1}{(s+1)(s+5)}}_{H(s)} \cdot \underbrace{\frac{1}{(s+7)}}_{X(s)} = \ldots = \frac{1}{24}\left[\frac{1}{s+1} - 3\frac{1}{s+5} + 2\frac{1}{s+7}\right]$$

$$y(t) = \frac{1}{24}\left(e^{-t} - 3e^{-5t} + 2e^{-7t}\right) u(t)$$

5) $$y[n] = 5\,\delta[n] + \left(\frac{5}{4} - \frac{25}{4}\cdot 0{,}2^n\right) u[n]$$

Alt: $$y[n] = \left(\frac{5}{4} - \frac{25}{4}\cdot 0{,}2^n\right) u[n-2]$$